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THE STRUCTURE OF THE La II SPECTRUM*

By W. F. Meggers

In 1914, Popow[1] published a composed triplet of La lines which he recognized from Zeeman effect observations as a ^3P ^3D combination. In the same year, Paulson[2] brought forth a list of constant differences occuring between La wave numbers but no attempt was made to interpret the regularities. Even earlier, Rybar[3] had made extensive measurements of the Zeeman effects in La spectra and many complex patterns, for which there was at that time no explanation, were published. An important contribution was made in 1924 by Goudsmit[4] who attempted an identification of spectral terms for La II by deriving g and j values from Rybar's observations. Twenty levels were found, but the l values for eight of these were undetermined; the Zeeman effects were interpreted to indicate that these 8 levels belonged to a spectrum of higher degree as conceived by Landé. Altogether, about 70 lines were accounted for by the 20 levels discussed by Goudsmit.

During the past two years the theory of spectral terms as developed by Heisenberg[5] and Hund[6] has given some suggestions as to the structure of this spectrum and new empirical data have become available. Among the latter are unpublished Zeeman effects by the late Professor B. E. Moore of the University of Nebraska, and a comprehensive description of La lines in regard to their behavior with temperature in

* Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce.

[1] Popow, Ann. der. Phys., *45*, p. 147; 1914

[2] Paulson, Ann. der Phys., *45*, p. 1203; 1914.

[3] Rybar, Phys. Zeit., *12*, p. 889; 1911.

[4] Goudsmit, Kon. Akad. Wet. Amsterdam, *33*, No. 8, 1924.

[5] Heisenberg, ZS. f. Phys., *32*, p. 841; 1925.

[6] Hund, ZS. f. Phys., *33*, p. 345; 1925.

the electric furnace by Dr. A. S. King and Miss Carter.[7] These new theories and facts justified another investigation of the structure of the La spectra, especially since an excellent wave number list based upon the measurements of Wolff[8] for wave lengths less than 5500A and data by Kiess[9] for the longer waves had been prepared several years ago when the structures of Sc and Yt spectra were being investigated[10] in this laboratory.

Doublets and quartets have been discovered among the arc lines but the analysis of the La I spectrum is not yet so satisfactory as that of the La II spectrum. In the present paper, therefore, only the terms and combinations characteristic of the spectrum of ionized lanthanum will be given; the structure of the La I spectrum being reserved for a future publication.

In addition to the 20 levels found by Goudsmit, 22 more have been found and the combinations of these 42 levels give the wave numbers of about 180 spark lines. The available Zeeman data, while not of the highest precision, have been sufficient to determine, without ambiguity, the spectroscopic character of each level. Singlet terms of S, P, D, F and G type have been established and triplet terms of P,

TABLE 1. *Relative terms in the La II spectrum*

Terms	Separations	Symbols	Combinations
00.0		$a^3\overline{F}_2$ VIII	a^1P, b^1P, $a^1\overline{D}$, a^1F, b^1F, a^3P,
	1016.0		b^3P, c^3P, $a^3\overline{D}$, $b^3\overline{D}$, a^3F, b^3F, $a^3\overline{G}$
1016.0		$a^3\overline{F}_3$	
1394.5		a^1D_2 D	a^1P, b^1P, $a^1\overline{D}$, a^1F, b^1F, a^3P,
	954.8		b^3P, c^3P, $a^3\overline{D}$, $b^3\overline{D}$, a^3F, b^3F, $a^3\overline{G}$
1895.1		a^3D_1 d_3	a^1P, b^1P, $a^1\overline{D}$, a^1F, b^1F, a^3P,
			b^3P, c^3F, $a^3\overline{D}$, $b^3\overline{D}$, a^3F, b^3F, $a^3\overline{G}$
1970.8		$a^3\overline{F}_4$	
	696.5		
2591.6		a^3D_2 d_2	
	658.8		
3250.4		a^3D_3 d_1	
5249.8		$a^3\overline{P}_0$ $\overline{p}_3$	a^1P, a^1D, a^1F, b^1F, a^3P, b^3P,
	468.4		c^3P, $a^3\overline{D}$, $b^3\overline{D}$, b^3F.
5718.2		$a^3\overline{P}_1$ $\overline{p}_2$	
	509.3		
6227.5		$a^3\overline{P}_2$ $\overline{p}_1$	

[7] King and Carter, in press.

[8] Wolff, Zeit. f. wiss. Phot., *3*, p. 395; 1905.

[9] Kiess, Sci. Pap., Bur. Stand., *17*, p. 318; 1921.

[10] Meggers, Jour. Wash. Acad. Sci., *14*, p. 419; 1924.
Meggers and Moore, Jour. Wash. Acad. Sci., *15*, p. 207; 1925.
Meggers and Kiess, J.O.S.A. & R.S.I., *12*, p. 417; 1926.

TABLE 1. *continued.*

Terms	Separations	Symbols	Combinations
7394.6		a^1S_0	a^1P, b^1P, c^3P, $b^3\bar{D}$,
7473.4		a^1G_4	a^1F, b^1F, $a^3\bar{D}$, $b^3\bar{D}$, b^3F
10094.9		b^1D_2	a^1P, b^1P, $a^1\bar{D}$, b^1F, b^3P, c^3P, $b^3\bar{D}$, b^3F
17211.9		a^3F_2	a^1D, a^1G, $a^3\bar{D}$, $a^3\bar{F}$
	1023.7		
18235.6		a^3F_3	
	979.0		
19214.6		a^3F_4	
20402.8		$a^3\bar{G}_3$	a^1D, a^3D, $a^3\bar{F}$
	929.0		
21331.8		$a^3\bar{G}_4$	
21441.7		$a^3\bar{D}_1$	a^1D, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
	664.4		
22106.1	951.2	$a^3\bar{D}_2$	
	431.3		
22283.0		$a^3\bar{G}_5$	
22537.4		$a^3\bar{D}_3$	
22683.7		a^3P_0	a^1D, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
	21.5		
22705.2		a^3P_1	
	541.8		
23247.0		a^3P_2	
24462.6		$a^1\bar{D}_2$ VII	a^1S, a^1D, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
24522.7		a^1F_3 VI	a^1D, a^1G, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
25973.4		$b^3\bar{D}_1$ V	a^1S, a^1D, b^1D, a^1G, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
26414.0		b^3F_2 IV	a^1D, b^1D, a^1G, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
	1414.9		
	423.6		
26837.6		b^3F_3	
27388.3		$b^3\bar{D}_3$ $\bar{d}_2$	
	927.1		
27424.0		a^1P_1 III	a^1S, a^1D, b D, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
	1727.9		
27545.8		b^3P_0	a^1D, b^1D, $a^3\bar{P}$, $a^3\bar{F}$
	608.8		
28154.6		b^3P_1 II	
28315.4		$b^3\bar{D}_2$ $\bar{d}_1$	
	1343.6		
28565.5		b^3F_4	
29498.2		b^3P_2 I	
30353.3		b^1P_1	a^1S, a^1D, b^1D, a^3D
31786.0		c^3P_0 p_3	a^1S, a^1D, b^1D, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
	375.0		
32161.0		c^3P_1 p_2	
	1043.4		
32201.1		b^1F_3	a^1D, b^1D, a^1G, $a^3\bar{P}$, a^3D, $a^3\bar{F}$
33204.4		c^3P_2 p_1	

D, F, and G type. The terms are listed in Table 1 where their relative values, on the assumption that $^3F_2=0$, appear in the first column, the separations of levels in triplet terms in the second, my symbols and those of Goudsmit in column three, and the term combinations summarized in the last column.

From the level-separations it is obvious that the interval rule is not fulfilled and it would not be surprising, therefore, if deviations from the rules for Zeeman-effects and for line intensities were also in evidence. Irregularities among the Zeeman effects were in fact, pointed out by Goudsmit and additional examples could be given in connection with the new levels which have been found. This phase of the discussion is, however, postponed until the analysis of arc and spark lines of La is more complete. Whether or not the intensity rules hold for La II it is impossible to state until quantitative energy measurements are made. Considering the wide range of wave-lengths covered by many of the multiplets, on account of large level-separations, it is indeed gratifying that King's intensity estimates are in fairly good agreement with the rules. Flagrant exceptions such as occur with the lines at 4522 and 6296A, for example, are explained by the accidental superposition of two lines which have not been resolved.

The spectral lines resulting from combinations of the terms are presented in detail in table 2. Symbols and relative values of the combining terms are printed at the upper and left margins, the observational data on spectral lines resulting from the term combinations being represented by wave length values in air, wave numbers in vacuum, and King's estimate of arc intensities and temperature class. A few lines not in King's list have been reported by other observers; their intensities are followed by initials of the observers, thus E & H = Exner and Haschek, K = Kiess. Lines which are permitted by the combination rules for l and j quantum numbers but have not been recorded in published tables of wave lengths are represented by wave numbers computed from the combining terms. The majority of these unobserved lines are in the red and infrared portions of the spectrum which have not been thoroughly investigated for spark lines, partly on account of the very troublesome presence of profuse bands in the longer wave lengths region. Practically all of the spark lines of temperature classes III and IV are classified, but there are a considerable number of lines of class V which are not yet accounted for. The majority of these probably belong to combinations of the second set of terms with a third set no levels of

TABLE 2. *Combinations in the La II spectrum*

	$^3\bar{F}_4$ 1970.8	$^3\bar{F}_3$ 1016.0	$^3\bar{F}_2$ 00.0
$b^3\bar{D}_3$ 28315.4	3794.78 (600 III E) 26344.56	3662.08 (50 IV E) 27299.15	3530.66 (20 V? E) 28315.22
$b^3\bar{D}_2$ 27388.3		3790.83 (600 III E) 26372.01	3650.19 (125 IV E) 27388.08
$b^3\bar{D}_1$ 25973.4			3849.03 (200 III? E) 25973.25
$a^3\bar{G}_5$ 22283.0	4921.77 (400 III E) 20312.23		
$a^3\bar{G}_4$ 21331.8	5163.61 (20 V? E) 19360.90	4920.95 (400 III E) 20315.63	
$a^3\bar{G}_3$ 20402.8	? 18432.0	5156.74 (30 V E) 19386.72	4899.90 (300 III E) 20402.88
a^3F_4 19214.6	5797.59 (50 III E) 17243.77	5493.42 (20 V E) 18198.57	
a^3F_3 18235.6	6146.55 (5 IV) 16264.79	5805.77 (50 III E) 17219.50	5482.23 (50 V E) 18235.70
a^3F_2 17211.9		6172.73 (5 V) 16195.81	5808.29 (15 IV ?) 17212.00
b^3F_4 28565.5	3759.08 (600 III E) 26593.96	3628.84 (125 IV E) 27549.19	
b^3F_3 26837.6	? 24866.0	3871.62 (300 III E) 25821.67	3725.06 (40 IV E) 26837.61
b^3F_2 26414.0		3936.22 (60 IV E) 25397.90	3784.80 (20 V E) 26413.97
$a^3\bar{D}_3$ 22537.4	4860.88 (100 III E) 20566.68	4645.31 (40 V E) 21521.06	4435.85 (6 IV E) 22537.30
$a^3\bar{D}_2$ 22106.1		4740.25 (150 III E) 21090.06	4522[a].38 (500 III E) 22106.07
$a^3\bar{D}_1$ 21441.7			4662.54 (200 III E) 21441.55
a^3P_2 23247.0		——— ? 22231.0	4300.44 (30 IV E) 23246.90
a^3P_1 22705.2			4403.03 (8d III A) 22705.26
b^3P_2 29498.2		3509.99 (25 IV E) 28481.97	? 29498.2
b^3P_1 28154.6			3550.82 (15 IV? E)— 28154.48
c^3P_2 33204.4		——— ? 32188.4	3010.80 (2 IV A) 33204.08
c^3P_1 32161.0			3108.45 (5 IV E) 32161.10

a. Also b^1D_2-b^1F_3

TABLE 2. *Continued*

	a^3D_3 3250.04	a^3D_2 2591.6	a^3D_1 1895.1
c^3P_2 33204.4	3337.48 (500 III E) 29954.17	3265.66 (150 III E) 30612.85	3193.01 (25? IV E) 31309.39
c^3P_1 32161.0		3380.88 (400 III E) 29569.61	3303.11 (200 III E) 30265.85
c^3P_0 31786.0			3344.55 (300 III E) 29890.87
a^3P_2 23247.0	4999.47 (200 III E) 19996.56	4840.00 (10 V E) 20655.40	——— ? 21351.9
a^3P_1 22705.2		4970.38 (100 III E) 20113.60	4804.03 (50 V E) 20810.05
a^3P_0 22683.7			4809.00 (80 V E) 20788.57
$b^3\overline{D}_3$ 28315.4	3988.52 (800 III E) 25064.87	3886.32 (400 III E) 25723.99	
$b^3\overline{D}_2$ 27388.1	4141.71 (120? IV E) 24137.82	4031.70 (300 III E) 24796.45	3921.54 (300 III E) 25492.98
$b^3\overline{D}_1$ 25973.4		4275.64 (60 IV E) 23381.76	4151.95 (300 III E) 24078.31
$a^3\overline{D}_3$ 22537.4	5183.41 (400 III E) 19286.96	——— ? 19945.8	
$a^3\overline{D}_2$ 22106.1	5301.98 (200 III E) 18855.63	5122.99 (150 III E) 19514.44	4946.43 (20 IV? E) 20210.99
$a^3\overline{D}_1$ 21441.7		5303.55 (100 III E) 18850.05	5114.56 (150 III? E) 19546.61
a^3F_4 19214.6	6262.30 (80? III) 15964.17		
a^3F_3 18235.6	6671.40 (6 IV? E) 14985.22	6390.45 (80 III) 15644.03	
a^3F_2 17211.9	? 13961.5	6837.91 (3 IV E) 14620.33	6526.98 (100 III E) 15316.81
b^3F_4 28565.5	3949.10 (1000 III E) 25315.10		
b^3F_3 26837.6	4238.40 (400 III E) 23587.21	4123.22 (600 III E) 24246.06	
b^3F_2 26414.0	4315.91 (12 V E) 23163.61	4196.55 (300 III E) 23822.43	4077.34 (400 III E) 24518.93
b^3P_2 29498.2	3808.74 (2) E&H 26247.98	3715.54(80 IV E) 26906.39	3621.74 (4 V E) 27603.18
b^3P_1 28154.6		3910.81 (15 IV? E) 25562.93	? 26259.5
b^3P_0 27545.8			? 25650.8

TABLE 2. *Continued*

	$a^3\overline{P}_2$ 6227.5	$a^3\overline{P}_1$ 5718.2	$a^3\overline{P}_0$ 5249.8
b^3P_2 29498.2	4296.05(200 IV E) 23270.67	4204.03(100 V E) 23780.00	
b^3P_1 28154.6	4559.30(50 V E) 21927.08	4455.82(25 V E) 22436.28	4364.68(30 IV E) 22904.76
b^3P_0 27545.8		4580.08(100 V E) 21827.59	
a^3P_2 23247.0	5873.99(3 IV) 17019.51	5703.29(10 III) 17528.88	
a^3P_1 22705.2	6067.15(IV?) 16477.64	? 16987.0	5727.30(4 V E) 17455.41
a^3P_0 22683.7		5892.66(1 V) 16965.56	
c^3P_2 33204.4	3705.83(125 V E) 26976.88	3637.16(50 V E) 27486.16	
c^3P_1 32161.0	3854.95(3) E&H 25933.34	3780.68(50 V E) 26442.77	3714.87(60 V E) 26911.18
c^3P_0 31786.0		3835.10(20 V E) 26067.60	
$a^3\overline{D}_3$ 22537.4	6129.55(15 IV?) 16309.90		
$a^3\overline{D}_2$ 22106.1	6296[b].10(50 IV E) 15878.48	6100.36(10 V) 16387.94	
$a^3\overline{D}_1$ 21441.7	6570.94(2n III) 15214.34	6358.13(5 IV) 15723.56	6174.19(1 V) 16191.98
$b^3\overline{D}_3$ 28315.4	4526.12(200 III E) 22087.82		
$b^3\overline{D}_2$ 27388.1	4724.38(6 V E) 21160.89	4613.40(200 V E) 21669.92	
$b^3\overline{D}_1$ 25973.4	5062.90(4 V E) 19746.05	4935.60(2 V E) 20255.34	4824.04(80 III E) 20723.76
b^3F_3 26837.6	4850.56(5 V E) 20610.44 ?		
b^3F_2 26414.0	? 20186.5	4830.50(1 V E) 20696.04	

b Also b^1D_2–$b^3\overline{D}_1$

	a^1S_0 7394.6	a^1D_2 1394.5	b^1D_2 10094.9	a^1G_4 7473.4
a^1P_1 27424.0	4991.26(20 IV E) 20029.43	3840.69(50 V E) 26029.63	5769.07(50 V E) 17329.02	
b^1P_1 30353.3	4354.41(80 IV E) 22958.80	3452.21(50 III E) 28958.65	4934.82(20 V E) 20258.50	

TABLE 2. *Continued*

$a^1\overline{D}_2$		4333.77(600 III E)	7058.09(20 IV E)	
24462.6		23068.13	14367.80	
a^1F_3		4322.51(150 III E)	?	5863.68(20 V E)
24522.7		23128.21	14427.8	17049.42
b^1F_3		3245.12(200 III E)	4522[c].38(500 III E)	4042.92(300 IV E)
32201.1		30806.74	22106.07	24727.65
a^3P_0				
22683.7				
a^3P_1	6529.76(4 III ?)	4691.16(30 V E)	?	
22705.2	15310.30?	21310.72	12611.3	
a^3P_2		4574.90(300 III E)	?	
23247.0		21852.29	13152.1	
b^3P_0				
27545.8				
b^3P_1	?	3755.85(20 IV E)	5535.67(15 V E)	
28154.6	20760.0	26760.11	18059.66	
b^3P_2		3557.26(20 IV? E)	?	
29498.2		28103.57	19403.3	
c^3P_0				
31786.0				
c^3P_1	4036.60(8 V E)	3249.36(100 III E)	4530.59(2 V E)	
32161.0	24766.35	30766.48	22066.04	
c^3P_2		3142.76(50 IV E)	?	
33204.4		31809.99	23109.5	
$a^3\overline{D}_1$	?	4986.82(100 III E)	?	
21441.7	14047.1	20047.30	11346.8	
$a^3\overline{D}_2$		4826.85(4 V E)	?	
22106.1		20711.69	12011.2	
$a^3\overline{D}_3$		4728.37(80 V? E)	?	6636.52(1 V)
22537.4		21143.04	12442.5	15063.98
$b^3\overline{D}_1$	5380.99(80 V E)	4067.40(125 IV E)	6296[d].10(50 IV E)	
25973.4	18578.79	24578.84	15878.48	
$b^3\overline{D}_2$		3846.01(20 V? E)	?	
27388.3		25993.61	17293.4	
$b^3\overline{D}_3$		3713.55(300 IV E)	5486.78(3 V)	4796.68(4 V E)
28315.4		26920.81	18220.49	20841.95
a^3F_2		6320.39(80 III)		
17211.9		15817.45		
a^3F_3		5936.22(12 V E)		?
18235.6		16841.08		10762.2
a^3F_4				8514.63(1) K
19214.6				11741.26

c Also $a^3\overline{F}_2$–$a^3\overline{D}_2$

d Also a^3D_2–$a^3\overline{D}_2$

TABLE 2. *Continued*

b^3F_2 26414.0		3995.77(600 III E) 25019.43	6126.08(10 V) 16319.16	
b^3F_3 26837.6		3929.22(500 III E) 25443.19	5971.10(1) K 16742.72	? 19364.2
b^3F_4 28565.5				4739.78(2 V E) 21092.15
a^3G_3 20402.8		5259.38(40 III E) 19008.37	? 10307.9	? 12929.4
a^3G_4 21331.8				? 13858.4
a^3G_5 22283.0				? 14809.6

	$a^3\overline{F}_4$ 1970.8	$a^3\overline{F}_3$ 1016.0	$a^3\overline{F}_2$ 00.0
a^1P_1 27424.0			3645.42(400 IV E) 27423.88
b^1P_1 30353.3			? 30353.3
$a^1\overline{D}_2$ 24462.6		4263.59(100 d, V E) 23447.85?	4086.71(400 III E) 24462.66
a^1F_3 24522.7	4432.96(2) E & H 22552.00	? 23506.7	4076.70(30 IV E) 24522.75
b^1F_3 32201.1	3306.98(10 IV E) 30230.39	? 31185.1	3104.58(60 IV E) 32201.14
	a^3D_3 3250.4	a^3D_2 2591.6	a^3D_1 1895.1
a^1P_1 27424.0		4025.89(40 IV E) 24832.23	3916.05(400 III E) 25528.76
b^1P_1 30353.3		3601.06(15 V E) 27761.71	3512.92(25 IV E) 28458.21
$a^1\overline{D}_2$ 24462.6	4712.90(12 V E) 21212.44	4570.97(8 V E) 21871.07	4429.90(500 III E) 22567.57
a^1F_3 24522.7	4699.62(20 V E) 21272.39	4558.48(250 III E) 21931.02	
b^1F_3 32201.1	3453.17(70 III E) 28950.62	3376.31(40 III E) 29609.64	

TABLE 2. *Concluded*

	a^3P_2 6227.5	a^3P_1 5718.2	a^3P_0 5249.8
a^1P_1 27424.0	4716.41(25 V E) 21196.64	4605.79(50 V E) 21705.74	4508.50(1) E & H 22174.13
b^1P_1 30353.3	? 24125.8	? 24635.1	? 25103.5
a^1D_2 24462.6	5482.23(50 V E) 18235.70?	? 18744.4	
a^1F_3 24522.7	5464.33(25 V E) 18295.42		
b^1F_3 32201.1	3849.03(200 III ? E) 25973.25		

which have as yet been identified. A systematic search for these is now being made.

The lanthanum atom has 57 electrons, three of which are in unfilled groups, namely two of 6_1 type (s) and one of type 5_2 type (d). The ionized atom has lost one s electron and the combined effect of the remaining s and d electrons, according to the Heisenberg-Hund theory, is to produce two spectral terms of low energy value, ^3D and ^1D. The configuration d^2 yields terms ^3F, ^3P, ^1S, ^1D and ^1G, also of low energy value and these together with ^3D and ^1D comprise the complete low energy set of theoretical terms. The middle set of terms originates with sp and dp configurations of electrons, the former giving rise to ^3P and ^1P and the latter to ^3P, ^3D, ^3F, ^1P, ^1D, ^1F. The outer structures of scandium and yttrium atoms resemble lanthanum and the regularities thus far observed in the spectra of Sc^+ and Yt^+ are accounted for by the above mentioned low and middle set terms. But in lanthanum some additional middle set terms have been found and to account for these electron configurations involving f type electrons are probably present. Thus, the configuration sf is expected to yield ^3F and ^1F terms and the configuration df will give the following set; ^3P, ^3D, ^3F, ^3G, ^3H, ^1P, ^1D, ^1F, ^1G, ^1H. The analysis is still incomplete, but five of these terms, namely ^3P, ^3D, ^3F, ^3G, and ^1F have been found; they establish the presence of configurations containing f-type electrons. The readiness with which these electrons come into play in producing prominent spectral lines is probably related to the rare-earth character of the lanthanum atom, since cerium and the remaining rare-earths differ from lanthanum only in the number of f-type electrons present.

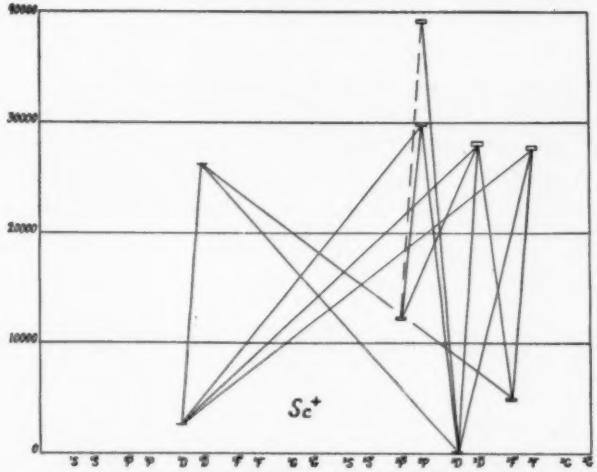


FIG. 1. *Spectral terms and combinations in the spectrum of* Sc^+.

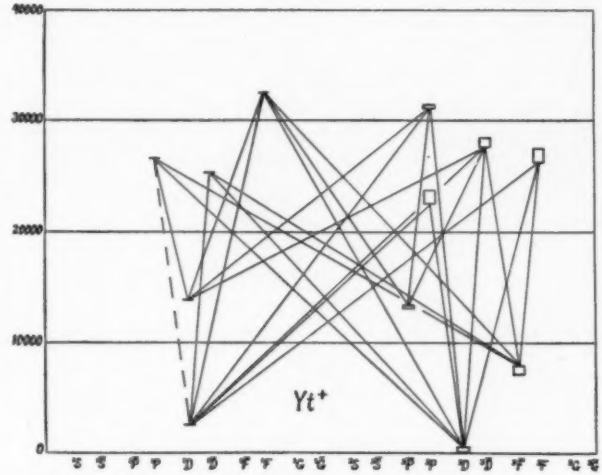


FIG. 2. *Spectral terms and combinations in the spectrum of* Yt^+.

In order that the resemblances and differences between the spectral structures of ionized scandium, yttrium, and lanthanum may be readily seen, a graphical representation of the known terms and combinations in these three spectra is reproduced in Figs. 1, 2 and 3. The terms ^3D and ^1D produced by the *sd* configuration are conspicuous in each case, but only in La II have all the terms originating with the d^2 configuration been found. Comparison of these lower terms in the three spectra shows that in Sc II and Yt II the *sd* arrangement easily gives the lowest levels and $^3D_1$ defines the normal state of these atoms. In La II, on the other hand, the terms from the d^2 configuration overlap those from *sd*, and $^3F_2$ describes the normal state of the ionized lanthanum atom. Among

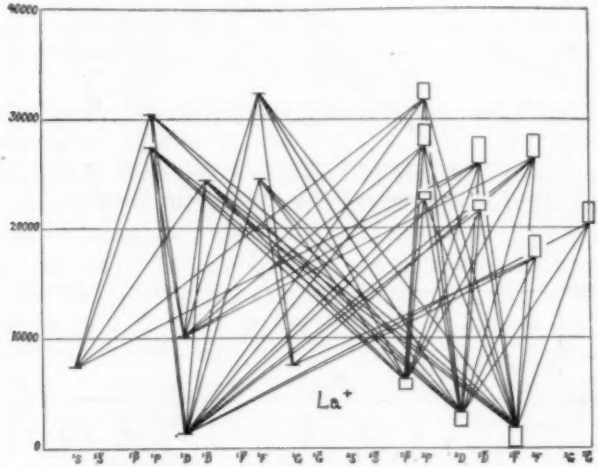


FIG. 3. *Spectral terms and combinations in the spectrum of* La^+.

the second set of terms the triad ^3P, ^3D, ^3F, accounted for by *dp* electrons, and the high ^3P from *sp* electrons are readily identified in the three spectra. In the La II spectrum all of the terms which can possibly arise from these configurations have been found, and the additional ones ^1F, ^3P, ^3D, ^3F, ^3G, are evidence for *sf* and *df* configurations, the latter giving the lowest terms of the second set. If configurations involving *f* electrons occur in Sc^+ and Yt^+ they are certainly much less conspicuous than in La^+.

Another respect in which the spark spectra of Sc, Yt and La show a remarkable difference is the strength of the combinations between singlet and triplet terms. The only intersystem combinations which have thus far been found in the Sc II spectrum are the following:

	$^1D_2$ 2540.8		$^1\overline{D}_2$ 26081.2
$^3P_1$ 29742.1	3675.26 (1) 27201.24	$^3D_1$ 00.0	3833.08 (3) 26081.31
$^3P_2$ 29823.9	3664.25 (1) 27282.97	$^3D_2$ 67.6	3843.06 (4) 26013.58
$^3\overline{D}_1$ 27917.7	——— ? 25376.9	$^3D_3$ 177.6	——— ? 25903.6
$^3\overline{D}_2$ 28021.2	3923.51 (2) 25480.19	$^3\overline{F}_2$ 4802.7	4698.30 (2) 21278.36
$^3\overline{D}_3$ 28161.0	——— ? 25620.2	$^3\overline{F}_3$ 4883.4	——— ? 20197.8
$^3F_2$ 27443.6	4014.49 (5 IV E) 24902.75		
$^3F_3$ 27602.3	3989.06 (2) 25061.50		

The relatively low intensities of these lines are apparent when they are compared with $^1D_2-{}^1\overline{D}_2$, 4246.82 A(100 III E), and combinations of the triplet terms.[11] In Yt II the intersystem combinations are considerably stronger[12] while in La II they include many of the spectrum's most powerful lines. This increasing prominence of intersystem combinations as spectra of corresponding elements in successive periods are compared is also observed in other parts of the periodic table but it is perhaps nowhere more striking than with the above mentioned spectra.

In a previous paper[13] it was pointed out that the strongest and most persistent or sensitive lines, spectrochemically, usually involve the normal state of the atom and a low excited state whose l value is one unit larger. Exceptions have been noted[14] in which these *raies ultimes* appear to involve a metastable state rather than the normal one when the quantum numbers (l, j) are larger for the former than for the latter. The empirical *raie ultime* of La II is the line $a^3D_3-b^3F_4$ with wave length 3949.10A. This is without doubt the strongest line in this spec-

[11] Meggers, Jour. Wash. Acad. Sci., *14*, p. 419; 1924.
[12] Meggers and Kiess, J.O.S.A. & R.S.I., *12*, p. 417; 1926.
[13] Meggers, Kiess and Walters, J.O.S.A. & R.S.I., *9*, p. 355; 1924.
[14] Meggers and Kiess, J.O.S.A. & R.S.I., *12*, p. 417; 1926.
Meggers, ZS. f. Physik, *39*, p. 114; 1926.

trum; King assigns it the value 1000, while the line $a^3\overline{F}_4 - a^3\overline{G}_5$ at 4921.77A is estimated as 400. If we may assume that the actual intensities are proportional to the squares of King's estimates[15] the line at 3949.10A is six fold more intense than the one at 4921.77A, although the latter not only involves the normal state of the ion but in addition has the largest l and j quantum numbers. This case is the first serious violation of the general rules governing *raies ultimes*, and it raises a question as to the relative importance of various factors which determine the distribution of energy in a complex line spectrum.

I wish to thank Dr. King and Miss Carter for kindly placing at my disposal their temperature classification of lanthanum lines, and also to acknowledge helpful suggestions and aid given by Professor H. N. Russell who has taken much interest in the analysis of this spectrum.

BUREAU OF STANDARDS,
OCTOBER, 1926.

Beyond the Milky Way. By George Ellery Hale, Honorary Director of the Mount Wilson Observatory. xv+105 pages. Charles Scribner's Sons. New York. $1.50.

The three chapters comprising this little book originally appeared as articles in Scribner's Magazine. The first chapter, entitled the ancestry of the telescope, discusses early astronomical instruments of the Egyptians, the Greeks and the medieval observers. The wonderful results of the researches at Mount Wilson, on stellar radiation, occupy a large part of Chapter II. The book receives its title from the last chapter, "Beyond the Milky Way," in which is a fascinating account, with many beautiful photographs, of nebulae and of the conclusions to which astronomers have been led as to the extent and structure of the universe.

The non-technical, yet exact, presentation which Dr. Hale has given makes the book of equal interest to both layman and specialist.

[15] Russell, Proc. Nat. Acad. Sci., *11*, p. 314; 1925.

ON TRANSFORMATION SPECTRA*

By E. L. Nichols, H. L. Howes and D. T. Wilber

The experimental work to be described in this paper will be considered in two parts: (a) A study of the color and appearance of various molten beads when subjected to changes of temperature; particularly when the beads contained traces of various metallic elements (or their compounds in solution). These observations, which had reference to the temperatures at which certain striking transformations occurred, were made by Nichols and Wilber during the summer of 1923. (b) A study of the spectra during these transformations when the elements introduced into the beads were of the group of the rare earths. This portion of the work was done by Nichols and Howes during the summer of 1925.

EXPERIMENTS OF 1923

This study had its origin in a desire to determine whether the striking effects described by Lenard[1] in 1905 are related to the selective radiation and luminescence from various incandescent oxides which we were then investigating.[2] Lenard finds that nearly all alkali salts when heated in the form of a bead in the Bunsen flame glow selectively. e.g. Cs and Rb salts, yellow green; K salts, stronger green; Na salts, sky blue; Li salts, pale blue.

The effect begins at a fairly high incandescence (a yellow heat) and at a temperature approaching the melting point of platinum, the colored glow goes over into white—in other words as in the case of the oxides whose incandescence we were studying in 1923 the selective radiation becomes non selective at high temperatures. These colors he found to be the same for a given metal, i.e. independent of the salt used except in the case of borates and phosphates where the phenomena are somewhat more complicated.

Lenard also studied spectrophotometrically, the continuous background of the spectra of the Bunsen flame containing various salts of Cs, Rb, K, Na, and Li. The distribution of intensities was found to be

* Received June 30, 1926.

[1] Lenard, Ann. d. Physik, *17*, p. 199; 1905.

[2] Nichols and Wilber, Proc. Nat. Acad. Sc., *6*, p. 693; 1920;
Physical Review, (2), *17*, p. 707; 1921.
Nichols and Howes, Physical Review, (2), *19*, p. 300; 1922.

distinctive for each metal and *independent of the acid radical.* Furthermore the color of the continuous light (obtained by reassembling the continuous background of the various flame spectra) was identical with the color of the corresponding beads. These observations are of great significance from the point of view of our own work.

APPARATUS

For the Bunsen flame of Lenard we substituted a hydrogen flame issuing from an orifice about .25 mm wide in a tube of nickel. The bead to be rendered incandescent was mounted from a loop of platinum wire or sometimes within a helical spiral of two or more turns. The helical mounting was used with salts which did not cling securely to a hot wire or when a greater depth of the molten material was desired. In the study of absorption spectra this device was especially advantageous because the faces of the bead perpendicular to the axis of the helix could be made flat and nearly parallel, giving an approximately cylindrical instead of a lenticular body. A V-shaped, Pt-PtRh thermoelement with the junction at the apex was so mounted that the tip

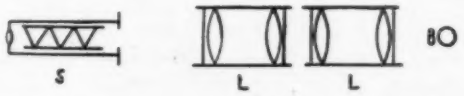


FIG. 1. *Direct vision spectroscope and lens system.*

was submerged within the mass of the bead. In some cases the bead could be hung to the thermojunction itself without other support; thus dispensing with the platinum loop or helix. The wires of this thermoelement were about 0.25 mm in diameter. If much thinner they are apt to melt off in the flame; if much thicker the bead does not reach the highest temperature desired.

While such thermoelements are ill adapted to the measurement of the temperatures of gases within a flame they will give quite accurately the temperature of a bead in which the junction is submerged, especially when the bead is molten. We found, for example, that when beads of LiF, RbF, NaF, NaCl, $NaSO_4$ and $Na_4P_2O_7$ were mounted in the flame with the junction submerged as above described, they fused when the temperature as indicated by the thermoelement reached the known melting point of the salt. The agreement indeed was surprisingly good; in general within one or two degrees centigrade.

Observations of the color of the bead and of the transformations which it underwent with change of temperature were made in a darkened room with the unaided eye or sometimes by the use of an enlarged image. Temperatures were taken with a potentiometer in the circuit of the thermoelement. To study the spectrum an enlarged image of the bead *B* was projected upon the slit of a hand spectroscope *S* (Fig. 1) by means of a system of large lenses *L, L*. With this device, instead of a spectrum of the light from the bead as a whole, a considerable amount of detail was obtainable and one could observe, for example, the spectra of separate particles floating around within the bead at times when the latter was beginning to cloud up or to clarify.

The adjustment of the apparatus was greatly facilitated by mounting the flame with a horizontal micrometer movement so that its position relative to the bead could be smoothly varied without disturbing the place of the bead in front of the lens system.

REPETITION OF LENARD'S EXPERIMENT

To see in how far Lenard's results could be obtained with our apparatus a bead of NaF was mounted in a platinum loop and the H flame was moved up laterally with the micrometer screw. As the temperature of the bead rose it became molten and clear, showed the blue color described by Lenard, then clouded with a red turbulent precipitate and finally began to glow with a white light which soon approximated the color and brightness of the platinum wire. On lowering the temperature there was a complete reversal of the phenomena.

Temperatures of the bead as indicated by the thermoelement were as follows:

Rising Temperature		Falling Temperature	
Blue appears at	995°C	Red cloud vanished	1210°C
Observation repeated	995°C	Observation repeated	1207°C
Red cloud appears	1205°C	Blue vanishes	990°C
Observation repeated	1210°C		

At 1287° the "white glow" of the bead approximated that of the platinum wire in color and brightness.

A naked junction in the outer layers of the hydrogen flame fluctuated between 1400° and 1500° with slight movements of the flame. When a bead about 2 mm in diameter was suspended from the junction, submerging it, the temperature fell to 1170°, at which point it remained quite constant. The combined heat losses by radiation and evaporation suffice to maintain the bead at a constant temperature two or three hundred degrees below the average temperature of the surrounding gases.

The spectrum of the bead was continuous, save for the Na lines, but the distribution was not that of a black body. A detailed spectrophotometric analysis would doubtless reveal details of structure, but to the eye it appeared to consist of a single broad band extending respectively: In the red phase, 700μ to .485μ (brilliant in red; green, bright; blue violet nearly absent); in the blue phase .550μ to .425μ (red, nearly absent; green, dim; blue violet, medium; crest at about .450μ).

Similar transformations, i.e. from blue to red were observed in a bead of NaCl (from Louisiana rock salt); but no quantitative determinations were made. Similar experiments with beads made from various salts of Li, K, and Rb served to confirm Lenard's statements as to the colors characteristic of those elements. It is of interest to note that the relation between color and the atomic weight is similar to that found by Tanaka[3] in the case of the fluorescence colors of certain solid solutions. More recently Nichols and Slattery[4] found a similar relation in the case of the fluorescence of solids in which a trace of uranyl served as an activator. Attention is called to this relation here because it affords a suggestion, of no obvious significance in itself, as to a possible explanation of the nature of certain effects in the incandescent beads to be considered later. Measurements of the temperatures at which these color effects occur in the various beads and of the phenomena preceding and following them bring out the fact quite clearly that the colors are definitely connected with transformations which take place within the molten material.

In most instances, the color effect, however, is only one of a more or less complicated series of changes which go on within the bead. The record in the case of LiF, for example, which was quite consistent on repetition both as to the temperatures at which changes occur and the

[3] T. Tanaka, J.O.S.A. & R.S.I. *8*, p. 294; 1924.
[4] Nichols and Slattery, J.O.S.A. & R.S.I. *12*, p. 449, 1926.

accompanying phenomena was as follows: The bead which when cold is clouded and almost white, turns black (as viewed by reflected light) at 642°. At 651° this change is completed and at 662° the bead clears up and becomes transparent. At 801° the bead fuses (the melting point as given in the tables of Landolt and Börnstein is 801° ± 15). At 900° turbulence sets in, opaque masses form within the bead which becomes clouded and remains so until at 1085° it again clears with a light blue color which suggests the opalescence of a moonstone. At 1310° the blue disappears. The blackening of the bead noted above in the case of LiF as occurring between the narrow limits 642° to 662° was observed also in beads of Rb_2SO_4 at 585° and of $BaCl_2$ for the much wider range between 350° and 680°.

The effect is of course merely an indication that the bead passes from complete transparency to a high degree of absorption of visible radiation and vice versa at the upper and lower limits. At higher stages of incandescence this condition would manifest itself by a non-selective glow of the sort characteristic of black body radiation normal to the temperature in question. That under illumination sufficient to overpower the emitted light such substances will appear black is obvious.[5] Color as described in the present paper, however, except in the case of beads at temperatures below the red heat, is the color of incandescence, i.e. the color of the bead as it appears in a dark room by virtue of its own emitted light.

With the alkaline salts the material for the study of the radiation from molten beads is virtually exhausted. $BaCl_2$, which has just been mentioned, forms a clear molten bead which is bright blue when first heated to about 1200° and there are probably other examples, but for the most part one must go to higher temperatures than are obtainable with a platinum wire in a hydrogen flame.

Having in mind however that in the domain of luminescence many solids which are non-luminescent when taken in bulk at ordinary temperatures become activators of luminescence when in extremely dilute solid solution and on the other hand that nearly all refractory oxides which are white or nearly so, radiate selectively at the lower ranges of incandescence (500° to 1200°) in a manner which we are compelled to describe as a type of luminescence, it seemed worth while to examine the radiation of beads in which a trace of various oxides or other salts had been dissolved. The solvent selected for this purpose was $Na_2P_4O_7$

[5] Nichols and Howes, l.c.

produced by heating Na_2HPO_4. Into beads of this composition salts of numerous elements were introduced and the effects of heating were noted. A bead of sodium phosphate, without a mixture, goes through its transformations and color changes like those described in the earlier paragraphs of this paper and it was for modifications in these effects that we proposed to look. The transformations and color effects with this sodium phosphate bead are fortunately quite simple, viz.

Bead Cooling

1320°C	Clear and colorless.
1278°C	Begins to cloud up.
1220°C	Uniformly cloudy but not opaque
1086°C	Begins to clear.
957°C	(Melting point) clouds up with a fine lemon yellow color.

After solidification the bead remains clouded, glass-like, colorless.

Here the only transformation accompanied by color is that at the melting point, and, as long since noted by Lenard, the color is not the blue characteristic of the other sodium salts.

The elements from whose presence in the bead results of interest might be anticipated were taken, for convenience in the following four groups: (See Table 1). (A) Well known activators of luminescence (Bi, Cr, Cu, Mn, U). (B) Elements whose white, refractory oxides are luminescent at temperatures of incandescence. (Al, Be, Ca, Mg, Si, Ti, Zr). (C) Elements notable as selective radiators. (Ce, Er, La, Nd, Th). (D) The rare earth elements (Dy, Gd, Ho, Sm). In addition to the above a cursory test was made of salts of the following elements. (Ag, As, Ba, Cd, Co, Fe, Ga, Ge, Hg, Mo, Ni, Pb, Sn, Sr, Ta, Tl, Va, W).

Of all the beads tested in the course of these experiments the only ones failing to show indications of transformation were those containing germanium, molybdenum, nickel and cobalt. On the other hand the most interesting and important effects were not brought out until the adoption of the method of flame extinction in 1925.

EXPERIMENTS OF 1925

In the summer of 1925 two of the writers, Nichols and Howes, returned to the study of the color effects in beads from a somewhat different point of view. It was clear from the observations of 1923 that the appearance of color was directly connected with reactions taking place at definite temperatures. In the meantime it had been shown that

certain of the rare earths in solid solution when bombarded by cathode rays yielded fluorescence spectra of an extraordinary character. The luminescence bands of these substances are so narrow and linelike that

TABLE 1. *Transformation temperatures of beads* ($Na_4P_2O_7$)

(A) Activators of luminescence

Element in solution	Temperatures of transformations	Color changes
Bi	957° melts; clear	dark red
	1046° cloudy	bright yellow
	clears	
	1220° cloudy (blue streamers)[1]	blue
	clears	
	1312° cloudy	bright yellow-red
Cr	957° melts; (red flash on sudden cooling)	
	1050° clear	faint blue
	1260° clouding up	ruddy
	1400° cloudy; very bright	white
Cu (in form of CuO)	957° melts (red flash on cooling)	red
	990° clouding	orange
	1122° clouded	bright yellow
	1212° clearing (blue streamers)	bright blue
	1235° very clear, colorless and dark	
Mn	957° melts	
	1060° cloudy	orange yellow
	1200° cloudy	ruddy
	1330° clears up	blue
U (1% of $UO_2P_2O_7$)	957° melts; cloudy	yellow
	1112° still cloudy	green
	1208° still green	
	1308° clearing	red
	1370° clear	bright red
Zn (in form of ZnO)	957° melts; cloudy	red
	1130° clearing (with blue streamers)	blue
	1170° clear	ruddy
	1300° clear	ruddy

[1] The effect designated by the term "blue streamers" consists of the formation of a moving cloud within the clear body of the bead. The color of the cloud is that of the blue of a finely divided medium and marks the transition from clear to cloudy or vice versa.

the spectra resemble flame spectra rather than the usual spectra of solids. To produce analogous spectra at temperatures of incandescence, if possible, and to compare these with the luminescence spectra in some

definite way was the purpose of these later experiments. The clear-cut and strikingly beautiful way in which, in 1923, the emission bands of

TABLE 1 (continued)

(B) *Elements whose oxides show luminescence at high temperatures*

Elements in solution	Temperatures of transformation	Color changes
Al (in form of A_2O_3 very dilute)	1009° melts (clouds on freezing, clears on melting)	
	1078° clouding	ruddy
	1296° clouded	ruddy
	1357° clearing (blue streamers)	blue
	1397° clear	
Be (1% BeO)	20° clear	pale blue-green
	720° clear, turning purple on heating	a deeper blue
	955° melts (on freezing a red precipitate with turbulence within the bead)	purple
	1045° ruddy color vanishing	
	1220° blue and clear	blue
	1287° brighter	
	1312° slightly cloudy	ruddy
Ca (trace of $CaSO_4$)	957° bead, which is clouded when solid, melts and clears	faint pink
	1275° no change until this, then turbulence	ruddy
	1300° clearer and deeper red	ruddy
Mg (trace of MgO)	1040° melts; clouded	ruddy
	1125° clouded, bright	yellow
	1300° clearing (blue streamers)	blue
	1397° clear	
Si (trace of SiO_2)	1000° melts; partly clouded	
	1220° clouds up completely	yellow-orange
	1353° clearing (blue streamers)	blue
	1400° clear	ruddy
Ti (trace of TiO_2)	957° melts	
	1062° clearing	very blue
	1220° clear	yellow
	1329° clouding	ruddy
Zr (trace of Zr-O)	1054° solid; clouded, bright	yellow-white
	1170° melts and clears	
	1191° white streamers	
	1300° clear	ruddy

neodymium had been observed to come out, when beads containing that element were removed from the flame and allowed to cool, suggested a promising procedure for the new attempt.

The apparatus depicted in Fig. 1 was mounted in a room well darkened and the bead to be examined was adjusted in the hydrogen flame with its image carefuly focused upon the slit of the spectroscope. An observer whose eye had been rendered properly sensitive by previous protection from light seated himself at the eyepiece, the bead was brought to its maximum temperature 1400° by increasing the sup-

TABLE 1 (continued)
(C) *Selective radiators*

Elements in solution	Temperatures of transformation	Color changes
Ce (trace of CeO)	955° melts; goes over from deep red to yellow on fusion 973° changing to green 1249° clears (blue streamers)	red yellow green blue
Er	1000° melts; clouded 1220° clearing (blue streamers) 1355° very clear; emission bands of erbium are finely visible at the lower temperatures.	yellow-white blue
La ($La(NO_3)_3$)	Cold opaque 1045° melts 1086° clearing 1286° clouded (blue streamers) 1312° very clear	white bright yellow blue
Nd (Oxalate)	957° melts; while solid is lavender 1260° very bright; clouded throughout; no sudden transformations; shows the well known emission bands of neodymium, especially well at lower temperatures	pale purple ruddy
Th $Th(NO_3)_2$	1094° melts 1135° clears 1220° blue is gone 1329° very clear	blue

ply of hydrogen and with the observer's attention fixed upon the spectrum the flame was suddenly extinguished (or reduced to a mere pilot flame.) As the bead passed through the temperature of transformation in general there was a more or less sudden *increase in brightness*. Sometimes this amounted to a veritable flash up, as in the case of lanthanum; sometimes it was scarcely perceptible when looking directly at the bead

itself. In the field of the spectroscope the effect sometimes amounted simply to a general momentary brightening of the whole visible spectrum, or a relative enhancement of some region, as the blue, green or red. Sometimes, when the bead contained a rare earth in solution there appeared in the rapidly darkening field and then faded away narrow line-like bands very like those that characterize the luminescence

TABLE 1 (continued)

(D) *The rare earth elements*

Elements in solution	Temperatures of transformation	Color changes
Dy	Cold, white like enamel	
	957° melts	
	995° clouding up	ruddy
	1125° clouded (bright)	yellow
	1220° clearing	
	1240° clear dark	
	1280° clear dark	
Gd	957° melts	
	1040° clear	
	1125° clouding up (blue streamers)	blue
	1260° clearing (blue streamers vanish)	
	1300° clear, bright	yellow
Ho	957° melts	
	995° clouding up	ruddy
	1040° clouded	red-yellow
	1175° clouded	yellow
	1220° clearing (white streamers)	
	1260° turbulent	ruddy
	1355° slightly cloudy (bright)	yellow
Sm	957° melts	
	1015° clear	ruddy
	1080° clouding up	yellow
	1175° clouded (bright)	yellow
	1340° clearing	red-yellow

spectrum of the element. There was something wierd about the apparition of these bands. Here we had the transformation spectrum for which we were looking. Once discovered the new effect became less and less obscure and with practice the bands could also be picked up on rising temperatures. With practice, too, it became possible to locate the bands by means of the wave length scale in the eyepiece of the spectro-

TABLE 1 (continued)

(E) *Miscellaneous beads*

Elements in solution	Temperatures of transformation	Color changes
Ag (traces of $AgNO_3$)	957° melts 1045° clear, dark 1125° slightly cloudy (blue streamers) 1280° Bead very clear and dark	 blue
As (trace of As_2O_3)	990° melts 1086° clouding 1220° cloudy 1312° clearing (blue streamers) 1396° slightly cloudy, bright	 yellow blue yellow
Ba (trace of BaO)	999° melts 1239° clear very red 1300° cloudy, bright 1320° cloudy, bright	 red yellow ruddy
Cd (traces of phosphate)	998° melts 1215° clouding (No striking color effects) 1240° clears again	
Cr Cr_2O_3 (trace)	957° melts (red flash on cooling) 1040° clear; very slight color 1260° clouding 1400° clouded; very bright	red blue ruddy white
Fe (trace of FeO)	Cold bead is gray-yellow becoming black on heating 950° melts 1040° opaque 1145° clearing 1300° clear (blue streamers in dark red bead)	 red orange-red orange-yellow orange-yellow
Ga (trace of gallium oxide)	Bead remains clear up to 1365° then clouds 1375° clears again	blue-white
Hg (trace of $HgSO_4$)	957° melts 1015° cloudy (red streamers) 1040° clear 1125° slightly cloudy, bright 1220° clear, dark, ruddy	 red yellow yellow ruddy
Pb (trace of PbO)	Cold bead, bluish gray 950° melts and clears; red before fusing 1125° clear, turning blue 1400° still clear	gray red blue purple

TABLE 1 (concluded)
(E) *Miscellaneous beads*

Elements in solution	Temperatures of transformation	Color changes
Sn (trace of SnO_2)	Cold bead is blue	blue
	957° melts	ruddy
	1100° clear, bright	yellow
	1173° clouding (blue streamers)	blue
	1312° clouded	red
Sr (trace of $SrCO_3$)	1027° melts	
	1045° slightly clouded	ruddy
	1220° clearing (blue streamers)	blue
	1380° very clear, very dark	
Ta	When cold bead is opaque	white
	957° melts; clouded	yellow
	1086° clearing	yellow-green
	1145° clear	blue
	1278° clear, very blue	blue
	1320° clear blue with ruddy tinge	blue-ruddy
	1329° clear, very blue	
Tl (trace of $TlCl_3$)	957° melts clear	
	980° clouding up	
	1125° clouded	yellow
	1220° clearing	ruddy
Va (trace of $VaCl_3$)	957° melts	
	1046° clear, dark	
	1102° very clear	green
	1126° clouding up	yellow
	1220° opaque	red-yellow
W (trace of oxide)	957° melts	
	1200° clouding up	
	1220° clouded (turns red)	red
	1268° very clear	

scope and thus to make a long desired comparison between the radiation of these substances at temperatures of incandescence with the luminescence due to cathode bombardment at ordinary temperatures.

LUMINESCENCE SPECTRA AND TRANSFORMATION SPECTRA

Thulium: When a trace of this element was put into solid solution in calcium fluoride and excited to luminescence by cathode bombardment a spectrum of narrow line-like bands was obtained. See Table 2

(fluorescence bands). When the transformation spectrum of a bead containing thulium was observed by the process of quenching already described the bands recorded in the same table (transformation bands) were noted. The two spectra have but one band in common, that at 1923, but in the sense in which that term is used in the study of luminescence spectra recently completed[6] they are "essentially identical." The principle here referred to may be stated as follows:

TABLE 2. *Transformation bands and fluorescence bands of thulium*

	Set A			Set B	
	$10^3/\mu$	Δ		$10^3/\mu$	Δ
Fluorescence Bands	1515		Transformation Bands	1667	
		111 = 7×16−1			256 = 16×16±0
	1626			1923*	
		98 = 6×16+2			15 = 16−1
	1724			1938	
		49 = 3×16+1			190 = 12×16−2
	1773			2128	
		64 = 4×16±0			
	1837				
		32 = 2×16±0			
	1869				
		272 = 17×16±0			
	2141				
		96 = 6×16±0			
	2237				
Transformation Bands		144 = 9×16±0			
	2381				

* Band 1923 is found in both the luminescense and transformation spectrum.

* In this and in the subsequent tables transformation bands are underscored; those appearing in both the fluorescence spectrum and the transformation spectrum are doubly underscored.

THE PRINCIPLE OF ESSENTIAL IDENTITY

(1) Two spectra are essentially identical when all their components are members of the same set or sets. (2) A set (as used here) is an aggregate of equidistant components where distance is measured in frequency units. (3) A full set is a set in which all the members, be-

[6] Nichols and Howes; see p. 99

tween the two outer limits of the spectrum are present as is the case in the luminescence spectra of the uranyl salts. (4) An incomplete set is one with missing members but in which distances between existing members are always exact multiples of the frequency interval. Sets in the spectra of solid solutions are in general incomplete. (5) The spectra to which this definition applies are spectra made up of bands either separate or over lapping. The structure in question has been found to occur even where the overlapping is so complete that components are entirely submerged and the spectrum appears to be continuous.

Europium: The emission of this element in borax is somewhat more complicated than in the case of thulium. Upon heating the bead the following effects are observed: At 820°C the bead glows *orange: clouded:* at 940°C the bead clears: at 1085°C the bead is clear; *green:* at 1145°C the bead is a brilliant green.

Cooling there is a distinct flash up of green at 1050° C. In the spectrum both red and green bands come out between that temperature and 920°, with crests at: .630μ(1587); .562μ(1779); .530μ(1887). At still lower temperature (760°) an orange band appears. This orange band is in two parts, a component, (a) narrow and sharp, at .601μ (1664) and a somewhat broader portion (b) with shaded edges and a crest at .608μ (1645). Component (a) is seen only when the flame is

TABLE 3. *Transformation bands and fluorescence bands of europium*

Transformation or fluorescence	Set *A*		Transformation or fluorescence	Set *B*	
	$10^3/\mu$	Δ		$10^3/\mu$	Δ
fluorescence only	1600		fluorescence only	1538	
		$18=1\times18\pm0$			$54=3\times18\pm0$
	1618			1592	
		$161=9\times18-1$			$18=1\times18\pm0$
Both	1779			1610	
		$36\times2\times18\pm0$			$35=2\times18-1$
fluorescence only	1815		transformation only	1645	
		$72=4\times18\pm0$			$19=1\times18+1$
Both	1887		both	1664	
		$55=3\times18\pm1$			$19=1\times18+1$
	1942		fluorescence only	1683	
fluorescence only		$107=6\times18-1$			
	2049				

suddenly quenched and disappears almost at once. Component (b) persists to much lower incandescence and can sometimes be seen on rising temperatures also.

Comparing these bands with the luminescence spectrum of europium we find: (a) That the green bands of the flash up at 1050° are members of Set *a* and both coincide with observed members of that set. (b) That crests of the orange bands at 760° belong to Set *b* of the luminescence spectrum, the component 1664 being coincident with an observed luminescence band of that set. The frequency distance to the other component 1664–1645 = 19 is within the errors of observation of these necessarily rough settings. See Table 3.

As to the band in the red seen at the higher transformation point and recorded as at .630μ, the chances favor the existence of a double crest with a frequency difference of 18 which being insufficiently resolved was taken to be a single crest of intermediate location. Whether this doublet belongs to Set *A* or Set *B* cannot be ascertained from the existing reading. An error of setting of five frequency units is unlikely. The existence of a band belonging to an entirely different system, perhaps to a different activator is by no means excluded.

Terbium: in borax develops several bands on cooling. The bead has a transformation at 900° characterized by a green flash up. Of the seven transformation bands which were located five are identical with actual fluorescence bands (see table 4), another occupies a vacant place in Set *A*.

The seventh, .530μ (1887) is found in the fluorescence spectrum of terbium, samarium, and europium but is probably due to the presence of a trace of europium.

Samarium in borax gave six bands during the transformation stages which could be positively located. There were indications of others which would probably be amenable to setting under more favorable conditions. Of those which we succeeded in placing three coincide with known fluorescence bands while all fall into the sets previously established for this element. (See Table 6). The observer at the eyepiece of the spectroscope announced the appearance of the green doublet .567μ—.572μ at a cooling temperature of 1050°. At 1000° these bands disappeared and were replaced by the red band with crest at .648μ which remained visible until the bead had cooled to 860°. By reference to Table 1 (D) it will be seen that within the former range a transformation of the bead from clouded to clear was recorded (color, yellow).

TABLE 4. *Transformation bands and fluorescence bands of terbium*

Set *A*			Set *A'*		
Transformation or fluorescence	$10^3/\mu$	Δ	Transformation or fluorescence	$10^3/\mu$	Δ
both	1739			1645	
		31=2×16 −1			64=4×16 ±0
	1770			1709	
		16=1×16 ±0	fluorescence only		15=1×16 −1
fluorescence only	1786			1724	
		16=1×16 ±0			304=19×16±0
	1802			2028	
		16×1×16 ±0			209=13×16+1
both	1818		both	2237	
		143=9×16 −1			
both	1961				
		47=3×16 −1			
transformation only	2008		band at 1887 is assigned to europium.		
		49=3×16 +1			
	2057				
fluorescence only		31=2×16 −1			
	2088				
		17=1×16 +1	set *B* of the fluorescence spectrum of terbium is not represented in the transformation spectrum.		
transformation only	2105				

Within the range, 1000°–860°, the bead congeals and the color is described as ruddy.

SUMMARY

(1) The observations of Lenard upon the colors of molten beads were confirmed and the temperatures at which the effects occur were determined. (2) Observations upon thirty-seven elements whose salts cannot readily be obtained in the form of molten beads were made by introducing a small quantity of a compound of the element to be studied into beads of sodium phosphate, or sometimes of borax. (3) Such beads heated in a hydrogen flame were found to undergo transformations, characterized by a sudden clouding or, vice versa a sudden clearing of the bead, which changes occur at definite temperatures. (4) These transformations are further marked by changes of brightness and frequently by a characteristic color which is present only during

TABLE 5

Transformation bands and fluorescence bands of samarium

Set *A*			Set *B*		
Transformation or fluorescence	$10^3/\mu$	Δ	Transformation or fluorescence	$10^3/\mu$	Δ
	1563			1475	
		161 = 10×16+1			64 = 4×16 ±0
	1724			1539	
		15 = 1×16 −1			15 = 1×16 −1
fluorescence only	1739		fluorescence only	1554	
		31 = 2×16 −1			63 = 4×16 −1
	1770			1617	
		32 = 2×16 ±0			47 = 3×16 −1
	1802		transformation only	1664	
					17 = 1×16 +1
				1681	
					31 = 2×16 −1
				1712	
					65 = 4×16 +1
			fluorescence only	1777	
					64 = 4×16 ±0
				1841	
					287 = 18×16−1
			both	2128	

Set *C*			Set *C'*		
	$10^3/\mu$	Δ		$10^3/\mu$	Δ
			both	1541	
					112 = 7×16 ±0
				1653	
					48 = 3×16 ±0
			fluorescence only	1701	
	1607				47 = 3×16 −1
		32 = 2×16 ±0	transformation only	1748	
	1639				16 = 1×16 ±0
		16 = 1×16 ±0	both	1764	
fluorescence only	1655				64 = 4×16 ±0
		17 = 1×16 +1	fluorescence only	1828	
	1672				
		159 = 10×16−1			
	1831				
		49 = 3×16 +1			
transformation only	1880				

the stage of transformation. (5) The effects are reversible; i.e. may be observed either during heating or cooling of the bead. (6) The spectra of the glowing beads are, in general, continuous or at least broad-banded. Where traces of erbium or of neodymium are used, however, the well known emission bands appear. (7) During transformation there is a brightening of the whole visible spectrum or of some special region (red, green or blue). (8) When the bead contains a rare earth element the transformation spectrum (best observed during cooling) consists of narrow bands. (9) These bands correspond to those of the cathodo fluorescence of the element as to location so that transformation spectra and fluorescence spectra are essentially identical.

CORNELL UNIVERSITY,
ITHACA, NEW YORK, OCTOBER, 1925.

Photometry. By John W. T. Walsh, Senior Assistant in the Photometry Division of the National Physical Laboratory. xxvii+505 pages. D. Van Nostrand Company, New York. $12.00.

This is one of the most important and comprehensive books on this subject which has appeared since Liebentahl's "Praktische Photometrie." As the author states in the preface: "The past fifteen years have been so fruitful in the development of this branch of technical physics that the present volume may not seem altogether superfluous."

The book contains sixteen chapters, viz.: Historical Note; Radiation; The Eye and Vision; The Principles of Photometry; Standards of Candle-power; Measurement of Candle-power in a Single Direction; Candle-power Distribution and Total Flux Measurement; Heterochromatic Photometry; Spectrophotometry; The Measurement of Color; Physical Photometry; Illumination Photometry; Measurement of Brightness and of Reflection and Transmission Factors; Photometry of Projection Apparatus; Stellar Photometry; The Photometric Laboratory. There are over three hundred figures and photographs, and numerous tables of important data bearing on photometry and on the properties of light sources.

One very commendable feature of the book is the exhaustive list of references to be found at the end of each chapter. It is unusual to find such a complete literature citation in a book written *in English*. Needless to say the student of the subject will appreciate the ease with which he can find the original material under discussion.

Special mention should also be made of Appendix II which contains a list of the most recent "official" definitions of the various photometric quantities.

The author has done a valuable service in bringing this subject up-to-date. And, because of the many applications of photometric methods in both pure and applied science, the book should be readily available to all who are engaged in research or testing.

ON THE RELATION BETWEEN TIME AND INTENSITY IN PHOTOGRAPHIC EXPOSURE*

By L. A. Jones, V. C. Hall, and R. M. Briggs

During the past three years four communications dealing with the failure of the reciprocity law on photographic materials have been published from this laboratory.[1] Thus far attention has been directed largely toward the establishment of a definite functional relationship between the various factors involved, the materials being developed under standardized conditions. The data obtained have led to some very definite and interesting conclusions although a complete and general solution of the entire problem is far from being attained.

Since the discovery some forty years ago that the reciprocity law was not valid many papers have been published on the subject. Wide disagreement exists between the conclusions drawn by various workers in this field and it seems desirable if possible to inquire into some of the causes of these discrepancies. An examination of the literature shows that each experimenter has employed some particular procedure in the development of the exposed materials. Practically as many developing agents and formulas have been used as there are different investigators of this subject. This suggests that perhaps the lack of agreement may depend in some way upon the way in which the latent image is converted into a visible deposit of metallic silver particles. In our previous communications very little work has been described on the influence of *developing agent* or *developer constitution* or *extent of development* upon the magnitude of the reciprocity failure. In this paper some results bearing on this phase of the work will be reported.

So far as the authors are aware the only reference in the literature to a possible influence of the developer upon the failure of the reciprocity law is from Kron's classical paper "On the Blackening Law of Photographic Plates."[2] Kron's work on this subject has been extended by

* Fifth Paper. Communication No. 292 from the Research Laboratory of the Eastman Kodak Company.

[1] Jones, Loyd A. and Huse, Emery, On the Relation between Time and Intensity in Photographic Exposure, J.O.S.A. & R.S.I., *7*, p. 1079, 1923, *11*, p. 319, 1923.
Jones, Loyd A., Huse, Emery and Hall, Vincent, J.O.S.A. & R.S.I., *12*, p. 321; 1926.
Jones, Loyd A. and Hall, Vincent, J.O.S.A. & R.S.I., *13*, p. 443; 1926.

[2] Kron, E., Uber das Schwarzungsgesetz Photographischer Platten, Pub. Astrophys. Obs. Potsdam. No. 67, 1913.

Halm[3] who used a modification of an equation suggested by Kron to represent his "curves of constant density" formed by plotting log It against log I for a constant density. The Kron-Halm formula is

$$It=\frac{I_0 d_0}{2}\left[\left(\frac{I}{I_0}\right)^{a}+\left(\frac{I}{I_0}\right)^{-a}\right] \tag{1}$$

Kron suggests that the rapid departure of his data from the straight line relationship required by the Schwarzschild equation in the region of optimal intensity may possibly be accounted for by differences in the action of various developing agents. Kron used rodinal developer in which the reducing agent is paraminophenol hydrochloride, while Schwarzschild used a pyro developer in his work.

Apparently Kron is the only worker who has done any careful work on the relationship between the extent to which a plate is developed and the magnitude of the reciprocity failure. His results indicate that the only effect of increasing the time of development is to increase the value of optimal intensity I_0. The magnitude of the change in optimal intensity was found by him to be considerably greater for materials of relatively low sensitivity. No change in the curvature, as indicated by the value of a (equation 1) was found by him in the case of any of four photographic materials studied.

EXTENT OF DEVELOPMENT

In one of our previous communications, the third paper of this series, preliminary data relating to the influence of the extent of development upon the magnitude of the reciprocity failure were presented. It was shown that the extent of failure, as measured by the variation of density with intensity for a constant exposure, increased greatly with development and that the dependence of gamma on intensity increased with increasing development time. Some further data have now been obtained relating to this point which are in substantial agreement with our previous conclusions.

Figs. 1 and 2 show results obtained on Par Speed Portrait film and Motion Picture Positive film and are representative of those obtained from experiments on other emulsions on plates and films. Fig. 1 shows constant density curves for Portrait film when developed for 5 minutes and 30 minutes. Optimal intensity shifts from 0.625 mc to 1.5 mc with this change and the a value of the curve best representing the experi-

[3] Halm, J., On the Determination of Fundamental Photographic Magnitude, Mon. Not. R. A. S., *75*; 1915.

mental points decreases from 0.24 to 0.21. The solid lines in Figs. 1 and 2 are plotted by use of the Kron-Halm equation using the most probable values of I_0 and a as determined from the experimental measurements. The points represented by the small circles are actual experimental determinations. The dashed portions of the curves at the low end of

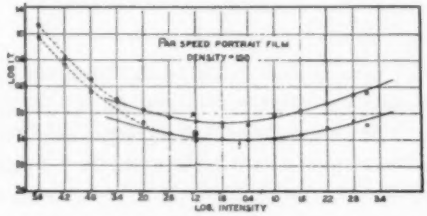


FIG. 1. *Constant density curves for Par Speed Portrait film. A, 5 minutes development; B, 30 minutes development. Pyro developer.*

the intensity scale illustrate the way in which the experimental measurements in this region of low intensity depart from the Kron-Halm equation. It will be noticed that the equation fits the data for a range of only 1–100 below optimal intensity but as far as the data extends on the high intensity side.

Fig. 2 shows similar results for Positive film. The failure for this material is somewhat greater than for Portrait film, below the intensity at which the experimental curve breaks from the catenary,

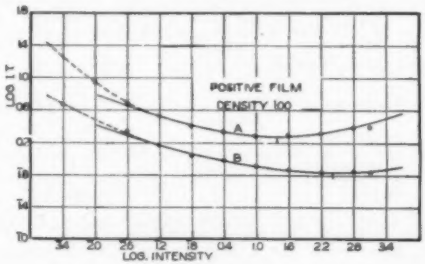


FIG. 2. *Constant density curves for Positive film. A, 5 minutes development; B, 30 minutes development.*

but in the region of optimal intensity the curvature is the same as for the faster material. Optimal intensity shifts from 25 mc to 250 mc, while the a value decreases from 0.24 to 0.20. These changes are representative of the results at all densities except those on the shoulder of the characteristic curve where the development causes a somewhat

greater failure of the reciprocity law due to the approach to the threshold value of some of the grains. Over the toe and straight line regions of the characteristic curve increasing the development time raises optimal intensity and in most cases slightly decreases the curvature of the curves of constant density in the region of optimal intensity.

EFFECT OF DEVELOPING AGENT

The developers used in this work were chosen so as to effectively cover the range of reduction potentials and at the same time to obtain information on those commonly used in sensitometric and practical work. The developers used, with the reduction potentials as given by Nietz[4] are tabulated in table 1.

TABLE 1

Reducing Agent	Reduction Potential
Ferrous oxalate	0.6
Hydroquinone	1.0
Paraminophenol hydrochloride	6.0
Pyrogallol	16
Elon	20
Amidol	35

These values may be said to represent the relative energy of the developers from a practical standpoint. The concentration of reducing agent used was M/20 for all except ferrous oxalate, and the other constituents were as follows:

Sodium carbonate	50 grams*
Sodium sulphite	50 "
Potassium Bromide	1 "
Water	1000 cc

* Sodium carbonate omitted when using amidol.

The use of a small amount of bromide reduces fog considerably and its only effect on the relative energy of the developers is to increase the differences shown in table 1, since the lower the reduction potential of a developer the more it is affected by the presence of free bromide in the solution.

[4] Nietz, A., The Theory of Development, Monograph No. 3 from the Research Laboratory of the Eastman Kodak Co., pp. 15–19.

The ferrous oxalate was made as follows:

A	Potassium oxalate	25 grams
	Water	75 cc
B	Ferrous sulphate	25 grams
	Sulphuric acid (conc.)	¼ "
	Water	75 cc

The developer was mixed immediately before using, taking 1 part of B to 3 parts of A. The sulphuric acid was added to solution B to retard the change of ferrous sulphate to ferric sulphate. By titrating B with an N/10 solution of $KMnO_4$ the concentration of ferrous ion was checked and as soon as any change occured a fresh solution was made up.

The first step in the use of the various developers was to develop a series of strips for different times from 1–20 minutes, in each developer. All of the strips received identical exposure at an intensity estimated to be near optimal for the emulsion in question. After reading the

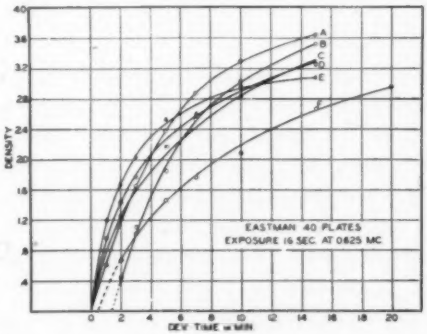


FIG. 3. *Density-time of development curves for various developers. Eastman 40 plates. A, pyro; B, hydroquinone; C, paraminophenol; D, amidol; E, elon; F, ferrous oxalate.*

densities and plotting the characteristic curves, density-time of development curves were plotted for each developer. A set of these is shown in Fig. 3, the material being Eastman 40 plates for an exposure of 10 meter candle seconds at 0.625 mc. These curves are quite close together but the definite differences due to the developing power are evident. Pyro, amidol, elon, and p.a.p. are quite close together for short developments but stand in the order named as to density in 15 min. Further development would probably not change the order. Hydroquinone is

slower to start developing, but develops rapidly after 2 minutes, and is second only to pyro after 9 minutes. Ferrous oxalate is of course very slow but continues to develop up to an hour or longer, so that the final density reached is about the same as for the organic developers.

For the purposes of these experiments a time of development was chosen so that a fixed exposure would develop to the same density with each developer. The conditions chosen for the three materials used are given in table 2.

TABLE 2

Material	Density	Int. mc.	Exp. mcs.	Development Times					
				FeC_2O_4	Hydro-quinone	p.a.p.	Pyro	Elon	Amidol
Eastman Slow Lantern	2.00	1.25	2.5		10′	10′*	5′30″	4′15″	4′30″
Eastman 33	1.60	1.25	1.25		7′	13′20″	4′30″	9′	5′30″
Eastman 40	2.40	.625	10.	12′	7′45″	6′45″	5′	5′20″	5′55″

* p.a.p. developed to D = 1.5 at 2.5 mcs.

As can be seen from the density-time of development curves for Slow Lantern plates there is a great deal of difference between the way the developers behave with different materials. The differences in reduction potential show up more with the slower materials, it being impossible to reach a density of 2.0 with p.a.p. on Slow Lantern at the exposure chosen unless development is carried so far as to produce excessive fog. A density of 1.5 was chosen for this developer and the data interpreted on the basis of a less complete development than in the case of the other reducing agents.

After determining the correct time of development complete runs consisting of about 20 sensitometric strips were exposed at each of 12 intensities over a range of from .00031 to 1280 meter candles. These were divided into groups of 3 strips from each intensity and one development made in each developer. These received identical treatment in fixing, washing, and drying, so that the only difference is in the developer used.

The diffuse densities of the strips were read on the Jones densitometer and averaged for each intensity and the characteristic curve plotted. From this curve the log E (log It) values for various densities were read and the curves of constant density drawn for the different developers.

Since the curves of constant density are quite similar for different densities the data can be compared for any density. The curves for that

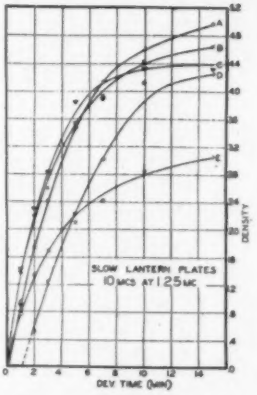


FIG. 4. *Density-time of development curves for various developers. Eastman Slow Lantern plates. A, pyro; B, amidol; C, elon; D, hydroquinone; E, paraminophenol.*

density chosen when determining the development time must intersect, at least at that intensity used in making the preliminary exposures. The data for Eastman 40 happen to be more inconsistent at the density chosen than for lower densities so that the curves for a lower density were used in making comparisons of shape. With this material the extent of development has but little influence upon the shape of the

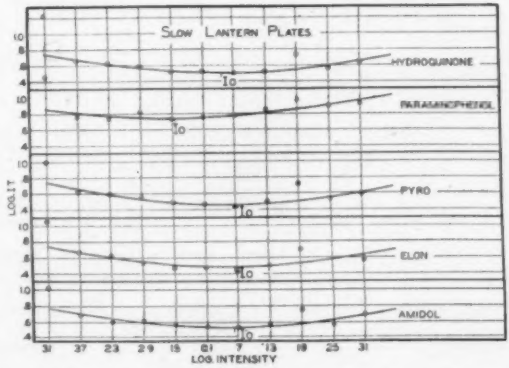


FIG. 5. *Constant density curves for Eastman Slow Lantern plates using various developers.*

"curves of constant density" hence the error introduced by using curves of lower density is negligible in comparison with those errors arising

from uncertainties in exposure and development. On Eastman 33 and Slow Lantern emulsions the curves shown are the ones for densities used to determine the time of development. Fig. 5 shows various curves for a constant density of 2.0 using Slow Lantern plates. All of these except for that of p.a.p. are identical within the limits of the experimental errors both as to position of optimal intensity and a value. As was pointed out the curve for the p.a.p. developer is for a density obtained by less complete development than in the case of the other reducing agents. The data shown in Figs. 1 and 2 indicate that the optimal intensity varies appreciably with the extent of development. It is probable therefore that the difference in the optimal intensity obtained with the p.a.p. developer is due to a difference in the extent to which development is carried rather than to any peculiar-

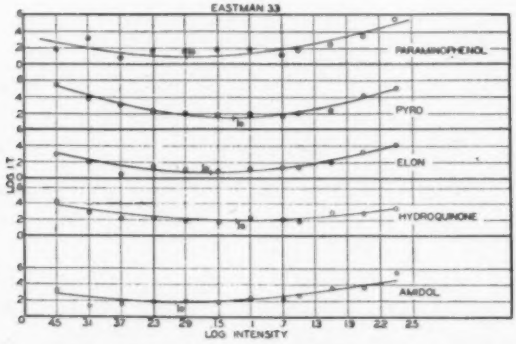


FIG. 6. *Constant density curves for Eastman 33 plates using various developers.*

ity of the reducing agent itself. The slightly lower value of a (.13 instead of .15) is probably within the limits of experimental error.

Fig. 6 shows similar curves for Eastman 33 plates. There is a definite difference shown between the developers but there is no systematic relationship between the developer characteristics, such as reduction potential, and the results, as pyro and hydroquinone are equally capable of giving high gammas, but show widely different curves of constant density, while amidol and hydroquinone, which have similar curves, represent the extremes of reduction potential as seen from Table 1. The results of p.a.p. are extremely irregular and no weight can be attached to them except to indicate that the results are of the same order of magnitude as for the other developers.

Fig. 7 shows similar results for Eastman 40 emulsion. These show some variation in the a value but the deviation from the mean is probably not beyond experimental error. The results for ferrous oxalate show exactly the same characteristics as those for the organic developers.

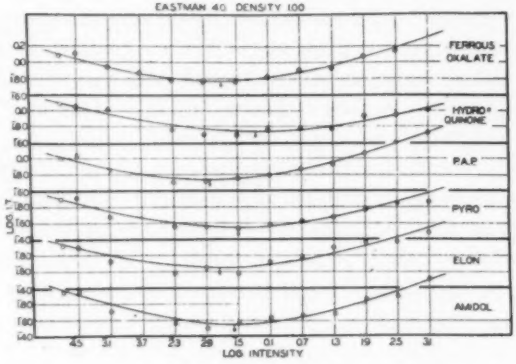


FIG. 7. *Constant density curves for Eastman 40 plates using various developers.*

There is therefore no evidence to indicate that the nature and magnitude of the reciprocity failure is any different for this inorganic developer than for a large group of organic reducing agents.

CONCLUSIONS

After a study of a very large number of emulsions of different types it seems safe to conclude that there is no silver bromide or bromo-iodide emulsion for which the curves of constant density in the region of optimal intensity may not be represented by the Kron-Halm equation

$$It = \frac{I_0 t_0}{2}\left[\left(\frac{I}{I_0}\right)^{a} + \left(\frac{I}{I_0}\right)^{-a}\right]$$

Several emulsions have been found which have optimal intensity so high that no resemblance to the curve represented by the above equation can be reached by intensities as high as sunlight, but in all cases where optimal intensity has been reached or exceeded the catenary type seems much superior to the hyperbola preferred by Kron. No reason for the many discrepancies in results reported by different workers has been found in a study of different developing agents. The only remaining point of difference is in the quality of the exposing radiation, the light used in all of this work being of color temperature 2500°K while that

used by Kron was considerably lower. The study of the effect of quality of incident light on the failure of the reciprocity law is now in progress.

The results of the work on the extent of development, checking previous data on the subject, show that the major effect of extending time of development is to shift optimal intensity to a higher value. If there is any change in the curvature it is slightly less at long developments than for short, but since the variations in value of a are of the same order as the experimental uncertainty it can not be said that any definite relationship between a and extent of development has been established.

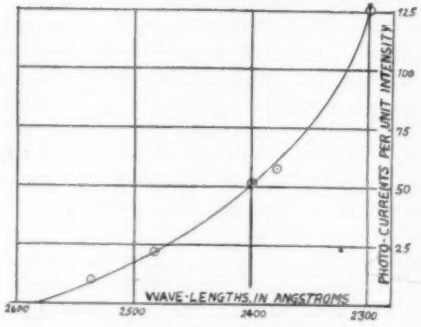
The effect of different developing agents on the magnitude and character of the reciprocity failure is smaller than might be expected considering the wide variation in developing strength and the general diversity of properties attributed to the use of certain developers. In fact the data available indicate that neither the reducing agent nor the constitution of the developing solution has any influence upon the shape of the constant density curve as expressed in terms of the value of a or upon the value of optimal intensity, I_0. We feel that the variation from these values as shown by the experimental data reported in this paper are within the limits of experimental error. This conclusion of course should be substantiated by much further experimental evidence. Definite proof that the character and magnitude of the reciprocity failure is independent of the reducing agent and developer constitution is highly desirable since it will simplify and facilitate the comparison of data derived under very different development conditions. It must not be forgotten however that the extent to which development is carried does have a marked influence upon the value of optimal intensity although it is exceedingly questionable at present whether or not this factor has any influence upon the value of a.

EASTMAN KODAK RESEARCH LABORATORY,
ROCHESTER, NEW YORK.
OCTOBER, 1926.

THE PHOTOELECTRIC THRESHOLD FOR GERMANIUM

By George B. Welch

The photoelectric threshold for germanium was determined by plotting the currents per unit intensity of monochromatic radiation against the wave lengths used, and extending the curve until it cut the wave length axis. The radiation intensities were measured with a sensitive thermopile connected to a Coblentz galvanometer. The specimen was prepared by polishing with emery cloth, and then placed in an evacuated cell charged sufficiently positively to insure saturation current. A Compton electrometer of sensitivity 1300 mm per volt measured the photocurrents, the rate of drift method being used. The effects of stray radiation were corrected by the employment of suitable filters.



Photoelectric threshold for germanium

The accompanying graph shows that the photoelectric threshold for germanium under the conditions given here is 2590A, a value which is probably accurate to within 25A. If we make use of the quantum relation $Ve = h\nu_0 = hc/\lambda_0$ where the symbols have their customary meanings and values, the equivalent value in volts of the energy required to release the photoelectrons from the surface is 4.77 volts.

Cornell University,
Ithaca, New York,
August, 1926.

Makers of Science—Mathematics, Physics and Astronomy. By Ivor B. Hart. 320 pages. Oxford University Press, London and New York. $2.00.

This work presents "some of the broader movements in scientific history so far as the physical and mathematical sciences are concerned" by giving, in connected chronological sequence, the biographies of some of those great figures in science, whose labors have made their respective subjects.

Centered around the biography of Aristotle, is a discussion of the science of ancient Greece. This is followed by the Alexandrian School: Euclid, Archimedes, Ptolemy. An excellent discussion is given of Roger Bacon and his times. Then come, in order, Copernicus, Kepler, Gilbert, Galileo, Descartes, Newton, Boyle, Ampere, Davy, Ohm, Faraday, and Kelvin.

To illustrate the method of presentation: The biography of Newton begins with a discussion of the status of science in the 17th century. Then follows a sketch of Newton's early life and education. In discussing his work in Optics, a brief account is given of Huyghens and of the rival theories of light. Newton's work on gravitation calls for mention of Hooke, Halley, Wren and others. Special attention is given to Newton's mathematical work and to "the battle of the calculus." A section on "Newton's later life" closes the account.

Scientists in general have grossly neglected the history of their respective sciences, and the biographies of the great figures whose memories are perpetuated by this, that or the other effect or law or apparatus. This volume, therefore, should serve a very useful purpose by placing before the busy (!) reader, within the compass of 300 pages, concise biographical accounts of some of those men whose names are the "household" words of physics, mathematics and astronomy.

Of course brevity dictated the omission of much very interesting material. But certain additions would have been very desirable. For example, one of the most important developments in the entire history of physics, namely the revival of the wave theory of light by Young, Arago and Fresnel in the early years of the 19th century is given hardly more than passing comment. Maxwell, one of the outstanding physicists of all time, is only mentioned incidentally.

Nevertheless, every student of the physical sciences should *read* a book of this kind—this volume is one of the best that has come to the reviewer's attention—and then place the book on the shelf for *ready* reference. [F. K. R.]

INSTRUMENT SECTION

THE DISTORTION OF SOME TYPICAL PHOTOGRAPHIC OBJECTIVES*

By A. H. Bennett

ABSTRACT

In connection with an examination of lenses to be used in airplane topographical mapping, a careful study has been made of the distortion of a large number of high-grade commercial photographic objectives. The apparatus designed for the measurement of distortion for infinite or finite object distance is described. The data show the distortion of the symmetrical anastigmat when the object is at an infinite distance, and also at unit magnification. At unit magnification, with a symmetrical lens, there should be no distortion, but tests of nominally symmetrical anastigmats show sufficient distortion to affect the performance of the lens. This distortion arises from lack of complete symmetry in the lens construction. Data for the distortion of the unsymmetrical anastigmat (Tessar type) are presented. The results show that rather large variations exist among individual lenses of nominally the same construction. The case in which the axis of the lens is bent, a result of slight prismatic action of the system, is discussed, and a method for determining the magnitude of tilt is described.

The importance of freedom from distortion of lenses used in airplane topographical mapping has been discussed by Gardner and Bennett.[1] In view of the increasing use of photographic objectives for such processes, a study has been made of the distortion of a number of symmetrical and unsymmetrical anastigmats. This work was begun at the request of Brock & Weymouth, Inc., Philadelphia, and the Army Air Corps, who have supplied the lenses on which the measurements reported have been made.

Distortion is defined as the linear distance from the position which a point image would occupy if the lens were perfectly orthoscopic to the actual position of the image. If the displacement arising from distortion is away from the center of the plate the distortion is positive. The amount of distortion is in general different for objects at different distances from the axis. Hence determinations of the magnitude of this defect must be made at different angular distances from the axis as far as the limit of the field of view.

Consider a lens possessing distortion to be rotated about an axis normal to the optical axis at the back nodal point for a paraxial bundle

*Published by permission of the Director of the Bureau of Standards, United States Department of Commerce.

[1] I. C. Gardner and A. H. Bennett. The Compensation of Distortion in Objectives for Airplane Photography. J.O.S.A. & R.S.I., *14*, p. 245; 1927.

of rays. If before the rotation the object point was axial, then after a rotation through an angle β the image of this object point is displaced in a direction normal to the line extending through object and nodal point by an amount $D \cos \beta$ (to a first approximation) where D is the distortion defined above. The measurement of this lateral displacement affords a convenient method for the determination of distortion.

The apparatus which we have specially designed for the application of the familiar method outlined above embodies many novel details of construction. The complete assembly of the apparatus employed for the measurement of distortion of a lens when the object is at an infinite distance is shown in Fig. 1. The essential parts are a collimator, A,

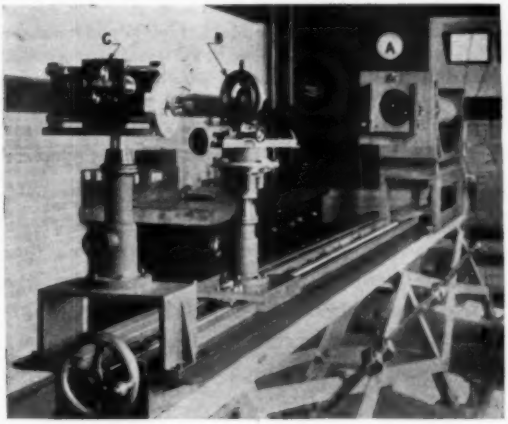


FIG. 1. *Apparatus for distortion measurements with infinite object distance.*

to give a beam of parallel light, the nodal slide, B, and a traveling microscope, C, all of which are mounted on an optical bench.

The objective lens of the collimator is of the usual two component type, well corrected for spherical aberration, and is 125 mm in diameter and of 1850 mm focal length. The large diameter of this objective permits measurements to be made on lenses of large linear aperture and those having entrance pupils at appreciable distances from the back nodal point. Correction for spherical aberration is an obvious requirement if all portions of the emergent beam are to be parallel. This condition is especially important in the testing of telephoto combinations in which the entrance pupil, located at a considerable distance from the back nodal point, is so displaced during the test as to receive

different portions of the beam. The collimator objective has a small amount of chromatic aberration which is of no importance in this work as a filter transmitting only a small band in the region of 580 mμ is used.

An illuminated vertical slit at the principal focus of the collimator is used as the source. Fig. 2 illustrates the housing of the slit together with the illuminating system. To focus the collimator a silvered optical flat is placed in front of the objective and an autocollimating method employed. For autocollimation the slit is provided with a small reflecting prism by which light may be conveniently introduced from the side. An eyepiece which can be focused on the slit is inserted and the collimator is focused in the manner of an autocollimating telescope.

The nodal slide shown in Fig. 3 is interesting as compared with the ordinarily less elaborate type because of the variety of adjustments

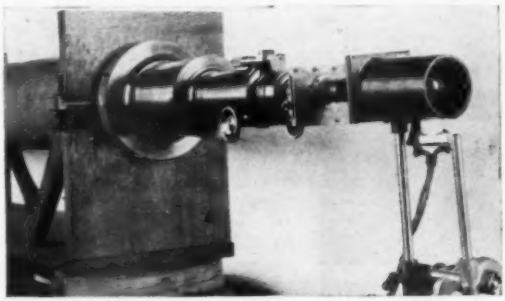


FIG. 2. *Focusing mechanism of collimator.*

with which it is provided. In use this has proved to be justifiable as it enables the correct placing of the lens to be accomplished with both speed and exactness. The lens is mounted in the universal chuck, *A*, the jaws of which are actuated by a rotation of the knurled collar surrounding the chuck. By means of the screw, *B*, the lens may be tilted vertically for adjustment, while a displacement transverse to the ways of the slide is effected by the head, *C*. The lens carriage can be moved along the ways by means of the lead screw, *D*, with which it can be engaged or released by a half-nut lifted or depressed by the small lever, *E*. These adjustments enable the lens to be brought into such a position that the vertical axis of rotation passes through the nodal point of the lens. The whole superstructure is mounted on a conical bearing which may be relieved from most of the weight through a rotation of the collar, *F*. This causes the load to be carried mainly by ball bearings and springs

in the cylindrical housing shown in the photograph (Fig. 3). The angular position of the lens is determined by the graduated circle and verniers, *G*.

The traveling microscope, used in measuring the shift of the image which may occur when the lens is rotated, is shown in Fig. 4. The microscope is fitted with an auxiliary tube sufficiently long to focus on a cross hair mounted over the axis of rotation of the nodal slide without mechanical interference. The microscope is supported by

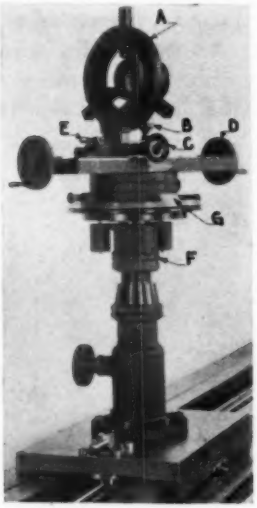


FIG. 3. *Nodal slide.*

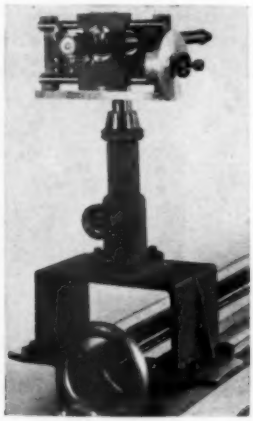


FIG. 4. *Traveling microscope.*

V-slides which are carried on carefully ground cylindrical rods. An examination of the screw shows that the periodic error does not exceed 0.004 mm.

Fig. 5 illustrates the disposition of the apparatus for the measurement of distortion for finite object distance. The collimator is dispensed with and a target consisting of two parallel vertical slits about 1 mm apart is used as the object. Both slits are used in the adjustment of the magnification to any desired value. The target is mounted on the micrometer slide, shown in the illustration, for convenience in adjustment. In the process of determining distortion at finite object distance the nodal slide and target are shifted along the bench until they are in such positions that the target and its image correspond to conjugate

points in object and image planes for the angle β through which the lens is rotated.

For the convenient adjustment of the various slides on the optical bench a lead screw extending the full length of the bench (274 cm) is

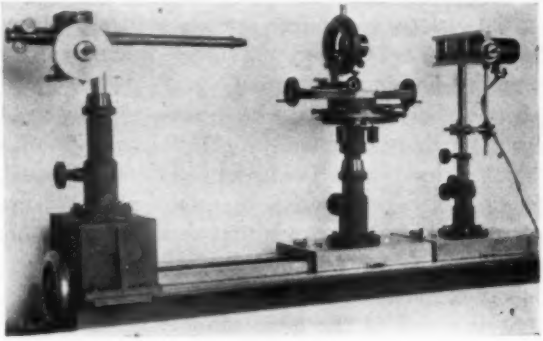


FIG. 5. *Assembly for measurement of distortion with finite object distance.*

provided. Any carriage can be connected to the lead screw by means of a half-nut which can be raised or lowered into contact by the lever on the top surface of the carriage (Fig. 5). This lead screw is particularly valuable in the measurement of the long focus lenses because the

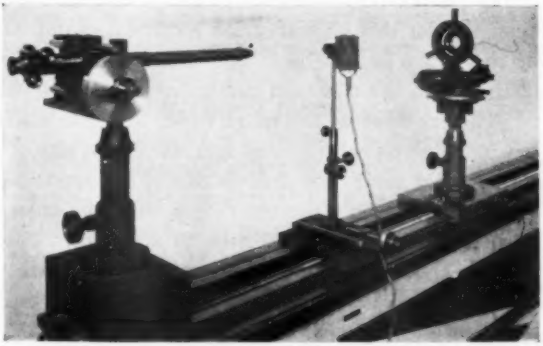


FIG. 6. *Device for locating axis of lens.*

observer with his eye at the microscope is enabled to secure an exact focus without the aid of an assistant.

For greatest accuracy in lens measurements in general it is necessary to adjust the axis of the lens parallel to the ways of the bench. This

adjustment is greatly facilitated by the use of a simple device (Fig. 6) which consists essentially of an illuminated diffusing surface about 5 mm in diameter, with a hole 1 mm in diameter at its center, and set at 45° to the axis of the lens. The observer sights through the small aperture and adjusts the lens until the images of the diffusing surface mirrored in the surfaces of the lens are concentrically placed. In this apparatus as constructed the diffusing surface is illuminated by a small lamp enclosed in a housing at one side.

Measurements of distortion have been made at 5° intervals over the useful field of a number of nominally identical lenses of both symmetrical and unsymmetrical types. The lenses had been purchased for use in airplane photography and precise measurements of distortion and focal length were desired in order that data from the photographs might be transferred to maps. The lenses were not specially made but were of highest commercial quality and were purchased in quantity in order that selection of the most suitable lenses might be made.

In the following tables the results of the measurements of the distortion of the lenses are reported to microns and it accordingly seems advisable to give the following discussion of the characteristics of the image on which the settings were made. The slit was so narrow that the width of the image is chiefly a diffraction effect. This is of course the narrowest image which the objective can produce. The microscope used in locating the position of the image had a magnification of about 150 diameters. Under this magnification a critical examination of the structure of the image is possible. The image consists of a series of diffraction bands with the effects of coma, curvature, and astigmatism superposed. The increase in the magnitudes of these aberrations at large angles from the axis results in an increase in the size of the image of the slit, while at the same time the contrast of the bands is decreased. This loss of contrast in the structure rather than the enlargement of the image causes larger accidental errors to arise when the measurements are made farther from the axis; for, if the brightest fringe is discernable, as is generally the case, the error in measurement is limited by the width of the fringe. In one instance the comatic flare at 30° from the axis extended 1.5 mm in a radial direction, yet the band which was by far the brightest and located at the extreme edge of the flare was about 15 microns wide. The center of the brightest band is taken as being at the true position. Using this criterion two observers arrive at results having no disagreement greater than that attributable to accidental errors, the magnitudes of which are discussed later.

A perfectly symmetrical system is distortion-free for unit magnification, i.e., when object and image are symmetrically placed with respect to the lens. This case is of interest when the lens is to be used for the copying and rectification of airplane photographs. Measurements show that the commercial symmetrical anastigmat departs by an appreciable amount from this ideal condition. This probably results from a small departure from symmetry introduced in manufacture. Table 1 gives the linear distortion in mm of several lenses of this type at unit magnification. In all cases the lenses were placed with the front surface as indicated by the manufacturer toward the object.

TABLE 1. *Distortion of symmetrical anastigmat at unit magnification*

No.	Equivalent focal length of lens (mm)	Distortion							
		Angular distance from axis							
		0°	5°	10°	15°	20°	25°	30°	35°
		mm	mm	mm	mm	mm	mm	mm	mm
1	149.3	.000	+.003	−.023	−.057	−.122	−.238	−.440	−.726
2	149.9	.000	.000	+.001	+.004	+.011	+.025	+.055	+.098
3	152.2	.000	−.002	−.006	−.013	−.023	−.047	−.094	−.181
4	188.1	.000	−.004	−.006	−.017	−.027	−.039	−.054	−.079
5	361.0	.000	+.001	+.003	+.007	+.012			

An inspection of the above table shows the necessity of taking into consideration the distortion of nominally symmetrical objectives at unit magnification. It is also shown that different lenses, nominally identical, differ greatly as regards distortion and that an individual test is necessary when lenses are to be used for mapping purposes. The distortion of the first lens of Table 1 is unexpectedly large and would vitiate the results of the measurements of a copied or transformed plate used in the process of topographical mapping.[2]

Table 2 gives the distortion of several symmetrical anastigmats but in this case the object is at an infinite distance. This corresponds to the condition when the lens is used for airplane photography. As before the lenses are nominally identical and in each case the front surface as designated by the manufacturer is toward the object.

Although a lens may have a small amount of distortion at a particular magnification, it does not follow that the distortion remains small when the magnification changes. This is illustrated in Fig. 7 in which the graphs show the amounts of distortion of a symmetrical anastigmat

[2] Gardner and Bennett, loc. cit.

with magnifications of one, two, and three diameters. The lens in this case had an equivalent focal length of 361.0 mm and its front surface was placed toward the image. The graph for unit magnification was

TABLE 2. *Distortion of symmetrical anastigmat with infinite object distance.*

No.	Equivalent focal length of lens (mm)	Distortion Angular distance from axis							
		0°	5°	10°	15°	20°	25°	30°	35°
		mm	mm	mm	mm	mm	mm	mm	mm
1	149.3	.000	+.002	+.004	+.009	+.014	+.024	+.055	+.153
2	149.7	.000	+.002	+.008	+.013	+.026	+.049	+.110	+.272
3	151.8	.000	+.004	+.012	+.025	+.049	+.088	+.167	+.328
4	152.1	.000	+.004	+.011	+.023	+.042	+.077	+.142	+.283
5	152.2	.000	.000	+.007	+.024	+.051	+.095	+.213	+.354
6	152.2	.000	+.004	+.010	+.028	+.058	+.114	+.218	+.411

carried to 20° only, because the measurements were taken to cover a certain size of negative which was to be copied and for which the above angle was sufficient.

Results of distortion measurements on a number of unsymmetrical anastigmats (Tessar type), with the object at an infinite distance, are

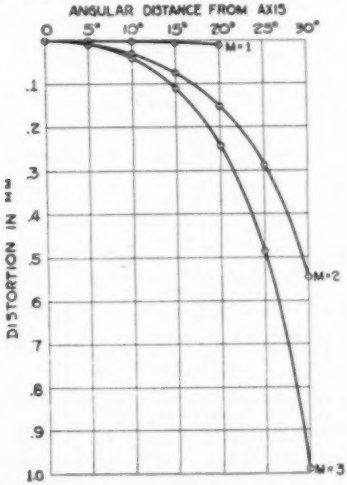


FIG. 7. *Distortion at various magnifications.*

given in Table 3. These lenses show satisfactory freedom from distortion out to 20° from the axis, and within this range excel the symmetrical lenses of Table 2. Beyond 20° the distortion of both types becomes

too large to be neglected where precise measurements are to be made on the photographic plate.

TABLE 3. *Distortion of unsymmetrical anastigmat with infinite object distance.*

No.	Equivalent focal length of lens (mm)	Distortion Angular distance from axis						
		0°	5°	10°	15°	20°	25°	30°
		mm	mm	mm	mm	mm	mm	mm
1	183.8	.000	+.002	+.007	+.011	−.017	−.122	−.335
2	183.9	.000	+.002	+.004	.000	−.020	−.120	−.324
3	184.0	.000	+.003	+.005	+.002	−.008	−.107	−.295
4	184.0	.000	+.001	+.004	+.007	−.002	−.088	−.255
5	184.1	.000	+.003	+.006	+.007	−.010	−.101	−.291
6	184.1	.000	+.002	+.006	.000	−.013	−.106	−.302

In general the accuracy of the results of distortion measurements varies with the angular distance from the axis at which the measurements are taken. This arises chiefly from the change in the appearance of the image in different portions of the field. It is necessary that the observer set the microscope on the brightest part of the image. At the larger angles the structure of the image unfortunately loses contrast and thereby necessitates the exercise of more judgment on the part of the observer, resulting in larger accidental errors in making the setting. As a typical example of the errors involved in distortion measurements made with the apparatus described, Table 4 shows the difference between extreme values, the average deviation from the mean, the probably error of the mean of five determinations of the distortion of a symmetrical anastigmat.

TABLE 4. *Errors in distortion measurements.*

	Angular distance from axis						
	5°	10°	15°	20°	25°	30°	35°
	mm	mm	mm	mm	mm	mm	mm
Difference between extremes	.006	.008	.012	.014	.016	.017	.031
Average deviation from mean	.0019	.0030	.0038	.0039	.0043	.0068	.0113
Probable error of mean	.0008	.0011	.0015	.0016	.0018	.0024	.0042

At times in measuring the distortion of a lens the effect of a bent optical axis was noted. Such an effect would be produced by adding a weak prism to a centered lens system. It probably arises from a slight

decentration of some of the components. This results in a shifting of the entire image in a radial direction. The magnitude of this shift may readily be determined along with the distortion measurements. A reading is taken with the traveling microscope set on the image of the collimator slit with the lens "squared on," i.e., the image lies in the center of the field of view. Readings are also taken with the lens rotated to each side by the same amount, say β. If the mid-point between these readings does not agree with the reading for the axial image, either the collimator is at an angle with the bench or the axis is bent. The effect of the former remains constant as the lens is rotated about its axis, but the latter causes the image to describe a circle. Hence if the displacement of the mid-point, mentioned above, from the axial position of image is

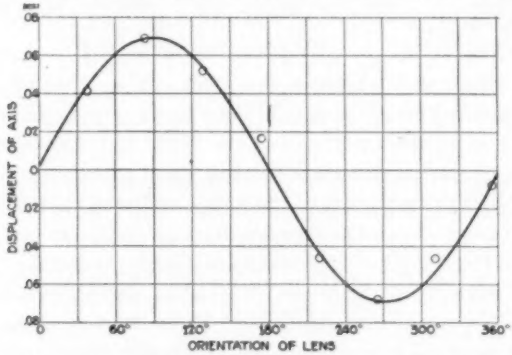


FIG. 8. *Graph illustrating measurement of angular tilt of optical axis.*

plotted against angle of orientation of lens in its own plane, a sine curve is obtained, illustrated in Fig. 8. In this figure the observed points are indicated by circles. The solid graph is an exact sine curve which serves for reference. The angle of bending of the axis is found by dividing the amplitude of the sine curve by the difference in length of oblique and axial chief rays. The example of Fig. 8 corresponds to a bending of the axis through an angle of 6′.3. Since the focal length of the lens is 241 mm, the image as a whole is displaced radially a distance of 0.4 mm.

In conclusion the writer wishes to acknowledge the numerous helpful suggestions of Dr. I. C. Gardner given throughout the course of this work, and the aid of Mr. F. A. Case in design of apparatus described, which was for the greater part constructed by Mr. H. C. Wunder and Mr. F. Knoop under the supervision of Mr. O. G. Lange in the instrument shop of the Bureau of Standards.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE COMPENSATION OF DISTORTION IN OBJECTIVES FOR AIRPLANE PHOTOGRAPHY*

By I. C. Gardner and A. H. Bennett

ABSTRACT

Lenses to be used in taking airplane photographs for topographic surveying must be substantially free from distortion. It is difficult to find objectives satisfactory in this respect and it is shown that in many cases the distortion can be compensated by a plane parallel plate used as an additional component of the lens. The effect of the plane parallel is generally not detrimental to the correction of the other aberrations of the objective. Experimental work is reported which justifies these conclusions.

Distortion-free, as applied to a photographic lens, is a relative term. For ordinary purposes the majority of lenses on the market meet the demands that are made upon them. For process work and copying a symmetrical double anastigmat is employed. It was formerly stated without restriction that such a lens is entirely free from distortion. Now however it is commonly known that even the symmetrical double anastigmat may be expected to show a moderate amount of distortion, except for the special case in which the object and image are symmetrically placed, i.e., when the object and plate are equidistant from the lens and the magnification is unity. But topographic surveying by photographs taken from airplanes demands exceptionally critical correction for distortion.

In such work the elevation of a point is determined by a measurement of the stereoscopic parallax. On at least one of the two exposures viewed stereoscopically it is necessary that the image of the point under examination be away from the center of the plate, and any displacement of this image, arising from distortion, if interpreted as stereoscopic parallax, will give an erroneous value of the elevation. Even if the distortion is known the introduction of a correction term is a tedious and time consuming operation. A lens having the greatest possible freedom from distortion is therefore desirable. The importance of this feature will be appreciated from the following illustration. Assume that one is using an 8×10 inch photographic plate and a 17 cm lens at a height of 5000 feet. The short dimension of the plate limits the field of view to approximately 60°. The exposures will be made as shown by Fig. 1. The point P is 10 feet above the plane AB. When the airplane

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is at S, directly over P, an exposure is made. For this exposure the image of P on the plate is the same as though it were not elevated above the plane AB. When the plane has advanced to the point T a second exposure is made where the angle $TD'B = 60°$. The image of P then lies near the edge of the plate. By reason of its elevation P will appear on the plate projected on the point D' where $D'C = 10 \text{ (tan } 30°) = 5.8$ feet. Now when the two plates are examined stereoscopically this displacement gives rise to a stereoscopic parallax which, correctly interpreted, gives the elevation of P above AB.

By simple proportion it can be shown that the displacement of the image of P on the photographic plate, which gives rise to this parallax, is 0.2 mm. In airplane mapping 10 foot contours are commonly desired

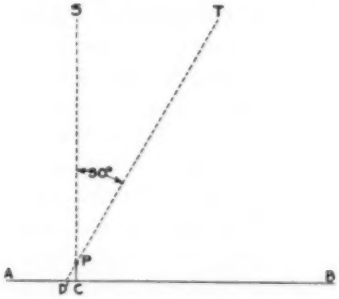

FIG. 1. *The origin of stereoscopic parallax in two successive photographs taken from an airplane.*

and even if an error as great as 5 feet in the elevation of a contour may be permitted, the total error in measuring the location of the image on the plate can not be greater than 0.1 mm. All of this tolerance cannot be allocated to the lens since a portion must necessarily be assigned to the apparatus by which the measurements are made. For the illustration at hand then, a lens with a 60° field and with maximum distortion certainly not greater than 0.1 mm is desired. If this cannot be obtained the alternative is to use a smaller field. There is a real gain here in accuracy, as the stereoscopic parallax varies as the tangent of the angular distance from center of field whereas the distortion varies as the cube of the angle. But with the smaller angular field one increases the amount of flying necessary and this greatly increases the cost of the field work. Consequently for airplane topography one wishes a lens giving the largest possible field free from distortion, and by "free from distortion" one means distortion certainly not greater than 0.1 mm, and much less if possible.

Bennett[1] shows not only that most photographic lenses have more than 0.1 mm distortion but that lenses of the same type and from the same maker, i.e., nominally identical, vary among themselves by more than this amount. For example, one symmetrical double anastigmat with a magnification of 1, the theoretical condition for no distortion, actually shows nearly 0.1 mm distortion at 30°. Whether this is because of the accidental asymmetries arising in manufacturing, or whether it is a minor lack of symmetry purposely introduced by the producer, is not known. For this type of lens with the object at an infinite distance, the case of interest in airplane photography, the distortion may be as much as 0.2 mm. Six lenses, supposed to be identical in type, gave a linear

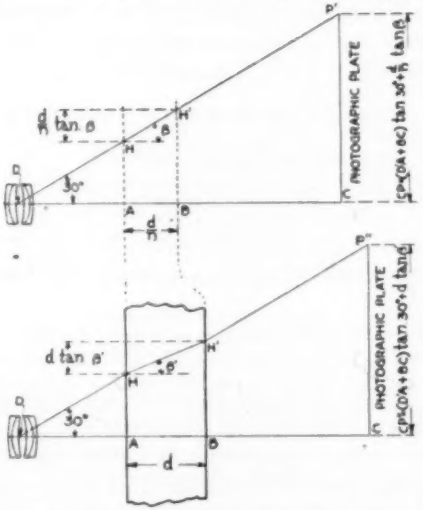

FIG. 2. *The manner in which a plane parallel plate introduces distortion. In first order imagery the layer of air between the two dotted lines in the upper diagram is equivalent to the glass plate in the lower diagram. But d/n tan β≠d tan β'. Hence P'' in the lower figure has a third order displacement with respect to P' in the upper figure, and this is the distortion.*

distortion at 30° ranging from +.06 to +.22 mm. The variation in distortion of different individual lenses of the same type is not unexpected. There are unavoidable slight variations in thickness and index from lens to lens. Accordingly it is the custom to assemble each lens separately and adjust the spacing by trial. But the criterion for this adjustment is sharpness of image in all parts of the field rather than the

[1] A. H. Bennett—The Distortion of Some Typical Photographic Objectives. J.O.S.A. & R.S.I., *14*, p. 235; 1927.

elimination of distortion, and hence the distortion varies from lens to lens.

Lenses can be and have been found which satisfy the requirements of airplane surveying as above stated but they are exceptional. However the use of a parallel plate to introduce compensating distortion, in the manner to be described, considerably lessens the difficulty in securing a suitable lens. The manner in which a plane parallel plate introduces distortion will be made clear by Fig. 2. The upper figure shows a chief ray proceeding from D', the exit pupil point of the objective, to the image at P'. In the lower diagram a plane parallel plate of thickness d and index n has been introduced between the lens and photographic plate. The portion of the ray path in air between the two dotted lines spaced d/n in the upper diagram corresponds to the ray path in the glass. CP'' minus CP' is the distortion and is given by the equation

$$\text{linear distortion} = d \tan \beta' - (d/n) \tan \beta \tag{1}$$

where β and β' are connected by the equation

$$n \sin \beta' = \sin \beta \tag{2}$$

If $\tan \beta'$ and $\tan \beta$ are expanded with all terms of the third order retained, and the relation of equation 2 introduced, equation 1 becomes

$$\text{linear distortion} = -\frac{d}{n}\frac{(n^2-1)}{(2n^2)}\tan^3\beta . \tag{3}$$

Substituting $n = 1.52$ it is found that for each millimeter in thickness the linear distortion introduced at 30° is .036 mm.

There are two ways in which this distortion may be utilized. If a camera lens having positive distortion is at hand, a plate of suitable thickness can be placed between the lens and plate to neutralize the distortion. This does not necessarily mean the introduction of an additional pair of glass air surfaces because, in making exposures, a filter is commonly used which, if given the proper thickness and placed behind the lens, may serve both as filter and compensating plate. The plate cannot be used to introduce positive distortion since the object is at so great a distance that the distortion introduced by a plate in front of the lens is negligible.

In some processes of airplane cartography the original negatives are copied in order to rectify them. In such a case the original exposure may be made with a lens not distortion-free and compensating distortion introduced when the copying is done. If the plane parallel plate is

placed between the negative to be copied and the lens, positive distortion is introduced. If the plate is placed back of the lens one secures compensation for positive distortion. Experimental results with the former case are shown in Fig. 3. In this case the negative was to be copied and enlarged two diameters. The plane parallel plate was placed in front of the lens to give positive distortion. The index of the plate is 1.57 and thickness 5.08 mm. This was not the correct thickness but was considered sufficiently near, and enabled a plate to be used which was available without special preparation. In substituting in equation 3 a factor 2 must be introduced, as the plate is on the object side of lens and the magnification is 2. The lens without the plane parallel plate gave negative distortion as indicated by the curve *B* in Fig. 3. The

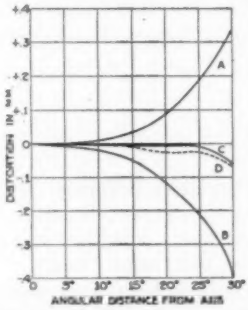

FIG. 3. *Graphs showing distortion. The shift of the image in millimeters arising from distortion is plotted against the angular distance from the center of the field. A. Computed distortion of plane parallel plate. B. Experimentally determined distortion of photographic objective. C. Experimentally determined distortion of objective and plane parallel plate combined. D. Distortion curve obtained by subtracting ordinates of curves A and B.*

computed distortion for this plate is represented by the curve *A*. The value of distortion for plate and lens combined, obtained by measurement on the optical bench, is indicated by curve *C*. If the ordinates of *A* and *B* are subtracted and the differences plotted, one obtains the dotted graph *D*, which is the distortion predicted for the combination of lens and plane parallel plate. The agreement between *C* and *D* is good. A portion of the deviation may be attributed to small errors of measurement, and the balance to irregularities in the plate which was not prepared especially for this test.

The remaining consideration is the effect of the plane parallel plate on the other aberrations of the lens. Computations by third order aberration equations show that the maximum diameter of the disc of

confusion at 30° from the axis for an aperture[2] $F/4.5$ resulting from spherical aberration, coma and curvature are 0.002, 0.02 and 0.3 mm respectively. These are the maximum values for points at the edge of the field. Except for curvature all these values are quite negligible. The broadening of a point image introduced by curvature is symmetrical about the point and hence not so serious as the asymmetric displacement introduced by distortion. Furthermore the effect of the plane parallel plate is to bend the image back from the lens. For a photographic objective the primary image surface at the edge of the field is commonly curved towards the lens. Hence in many cases the plane parallel will actually improve the image at the edge of the photographic plate by flattening the field. Exposures have been made with the plane parallel plate in combination with the lens and it is found that the aberrations introduced do not seriously impair the definition.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

An Introduction to Physical Science. By Ivor B. Hart. xii+306 pp. The Oxford University Press. London and New York. $1.50.

A beginner's book in physics in which an attempt is made to strike a balance between experiment and theory. The major part of the work is devoted to mechanics, heat and sound; light, electricity and magnetism comprising about 20 per cent of the book. The development is mainly by means of experiments and their applications, with examples and questions. There are some 200 well chosen cuts.

[F. K. R.]

[2] Nominally the lens is assumed to be F/4.5. When used for a magnification of 2 the aperture is F/13.5 and this value is used in the computations.

THE CASE "THALOFIDE CELL": ITS USE IN SPECTROSCOPY*

By Joseph Kaplan

In 1917 T. W. Case[1] found that fused thallium sulfide showed a slight change of resistance under the influence of light. After several years of research upon this substance Case succeeded in making a very sensitive cell which he called the "Thalofide Cell" to indicate that the sensitive substance was composed of thallium, oxygen and sulphur. The experiments which are described in the present paper were undertaken in order to study the sensitivity of the thalofide cell between wave lengths 5000 A and 12000 A and to compare these cells with other instruments which are used in this range for the measurement of radiation, such as the thermopile. Several possible uses for the cell are also suggested.

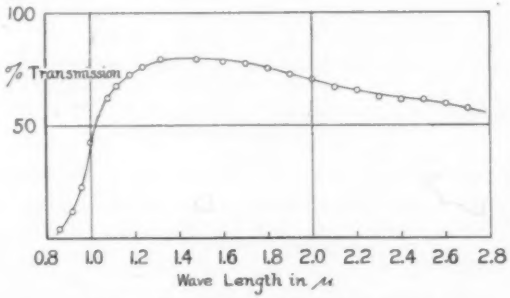

Fig. 1. *Transmission of black glass screen.*

The thalofide material is fused on a disc of quartz about three-fourths of an inch in diameter and is mounted in an evacuated glass tube. The effect of the vacuum, according to Case, is to increase the sensitivity of the element and also to preserve its life by preventing oxidation. In some of the present experiments the cell was screened from daylight by means of a black infrared transmitting glass of which the transmission curve is shown in Fig. 1. This was done because Case found that after a strong exposure to blue and to violet light, the resistance of the cell was somewhat permanently lowered and the recovery period after exposure was slow. The operation of changing cells was carried

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[1] Physical Review, *15*, p. 289; 1920.

out in a photographic dark room although this was a greater precaution than was absolutely necessary. Tests on about thirty cells under the same conditions of illumination showed their resistances to vary from $50 \cdot 10^6$ ohms to about $250 \cdot 10^6$ ohms. The sensitivities of these cells, when light of wave length 10000 A was allowed to fall on them, showed nearly all the cells to have about the same sensitivity, although occasionally an extremely good cell was found. Apparently there was no simple relation between the relative resistances of various cells and their sensitivities to light.

The apparatus which was used in the spectroscopic study of these cells consisted of a Hilger infrared spectometer with a 60° glass prism and a Hilger thermopile. A specially constructed attachment consisting

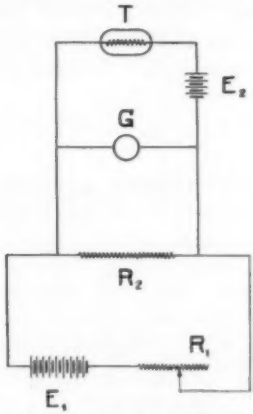

FIG. 2. *Electrical circuit used with the Thalofide cell.*

of a platinum mirror and a slit, made it possible for the light to shine alternately on the cell and the thermocouple, the same wave length drum being used for both. The source of light was a 1000-watt tungsten motion picture lamp of which one filament was allowed to shine on the slit. Since the region in which the cell is sensitive does not extend above 1.2μ it was found possible to replace the rock-salt prism by the glass prism. The prism was placed on the spectrometer table so that the entire motion of the wave-length drum was just sufficient to cover the range from 0.5 to 3μ. With the rock-salt prism this motion corresponded to a range of 10μ which indicated that the average dispersion of the glass prism was about three times as great as that of the rock-salt prism. For the spectral range from 0.5 to 3μ the glass prism is therefore

much superior to the rock-salt prism, and in addition of course does not deteriorate as rapidly.

In certain portions of the experiments it was found necessary to balance out currents of the order of 10^{-6} amperes from a Leeds and Northrup Galvanometer which had a sensitivity of $6 \cdot 10^{-11}$ amperes per mm deflection at 1 meter scale distance. This was done in a simple and safe manner by using the circuit shown in Fig. 2, in which T is the Thalofide cell and G the galvanometer. The resistance of the galvanometer was 570 ohms and for that reason care had to be taken that the galvanometer was not shunted by resistances small in comparison with the galvanometer resistance. Fortunately the most convenient values of E_2, Fig. 2, were such that all of these resistances were large.

A typical set of values of the parts of the circuit of Fig. 2 with the cell in the dark is given in Table 1.

TABLE 1.

$E_1 = 0.12$ volts	$R_1 = 30648$ ohms.
$E_2 = 96$ volts	$R_2 = 20000$ ohms.

Resistance of Thalofide cell about $1.5 \cdot 10^6$ ohms.
Resistance of galvanometer 570 ohms.
Sensitivity of galvanometer, 1 mm deflection at a distance of 1 m corresponds to $6 \cdot 10^{11}$ = amperes.

The transmission curve of the infrared transmitting glass, which was used to shield the cell from visible radiation, is shown in Fig. 1. Using this glass as a screen the spectral sensitivity curve of the thalofide cell was measured and is shown in Fig. 3. For this purpose slit widths of $\frac{1}{4}$ mm (corresponding to a wave length interval of about 0.03 at 1μ) were used and the galvanometer was a Paschen galvanometer of sensitivity 10^{-8} amps and resistance 10 ohms. The abscissas are wave lengths and the ordinates are the ratios of the galvanometer deflections when the light is allowed to fall on the cell to those when the light falls on the thermopile. It is seen from the curve that at 1μ the cell is nearly four times as sensitive as the thermopile. Taking into consideration the fact that the glass used as a screen absorbs more than 50% of the incident light at 1μ, the cell is eight times as sensitive as the thermocouple.

The action of the cell toward light is very satisfactory. On first making the current through the cell a few seconds elapse before the cell attains a steady value of resistance. Under the conditions of the experiments described in the preceding paragraph the sensitivity of

the cell was such that one could work in ordinary daylight without difficulty. When light falls on a cell or when the light is suddenly screened from the cell, the action of the cell is very rapid but with the intensities from a 1000 watt lamp and a slit width of a $\frac{1}{4}$ mm the cell required two or three minutes to come to equilibrium. However, no matter how long the time necessary for equilibrium, the cell invariably reached the *same* point of equilibrium. As the intensity of the incident light was made smaller and smaller so that finally the slits were about 0.04 mm wide, the time required for the cell to come to equilibrium also became smaller and smaller. With the very small intensity from a slit 0.04 mm wide the response of the cell was quick and steady. This of

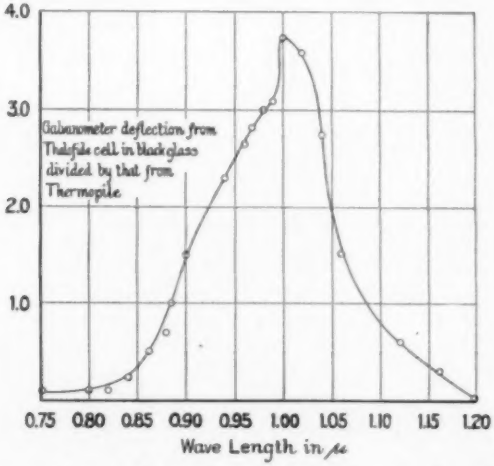
4.0
3.0
2.0
1.0
Galvanometer deflection from Thalofide cell in black glass divided by that from Thermopile
0.75 0.80 0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20
Wave Length in μ

FIG. 3. *Comparison of the Thalofide cell and the Hilger thermopile.*

course, makes the cell especially good for spectrophotometry where the light intensities are usually small.

In lining up the cell with a source of light, e.g. a slit, it is necessary to place the cell so that the area of sensitive material covered by the light is large. A very intense spot of light falling on a small area of sensitive material may not cause as great a change in resistance as a less intense spot covering a larger area even though the product of the intensity times the area is the same in the two cases. The reason for this is the distribution of sensitive material in the cell. The material was deposited in narrow strips on the quartz disc and unless care was taken in lining up the cell, much of the light was wasted on the region of the disk not covered by sensitive material. It was found, however,

that with narrow slits the optimum adjustment of the cell was accomplished with ease.

When the sensitivity of the cell and circuit were augmented by increasing the voltage E_2 of Fig. 2 certain difficulties appeared which required some care to surmount. With the increased sensitivity there was a steady drift of the galvanometer due to a steady change in resistance of the cell even at night. At first it was thought that the room was not dark enough and that the drift was due to changing illumination outside. The steady drift indicated such a possibility. One night, however, the cell behaved beautifully, having no drift and it was very easy to get a set of readings on some of the absorption bands of

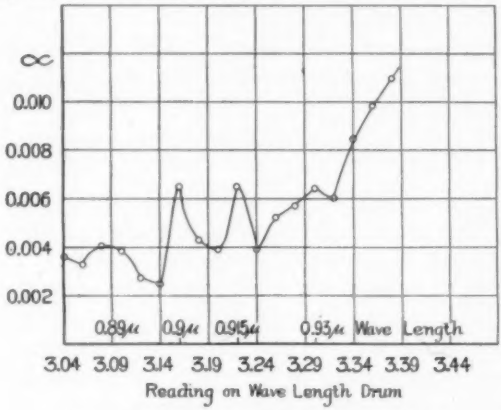

FIG. 4. *Fine structure of the* 0.9 mμ *water band.*

liquid water. The cell acted satisfactorily all during the next day. The drift started again after the cell was exposed to the light from a small tungsten lamp about one foot away. The experiment was tried several times and after each exposure a steady drift would occur which lasted for several hours indicating that the unsteadiness of the cell persisted for this length of time. The cell finally became steady, however, and remained remarkably so, for a deflection of one mm of the galvanometer corresponded to a change in resistance of 6000 ohms or 0.006 per cent of the resistance of the cell. How far the sensitivity can be pushed obviously depends on how steady the resistance of the cell remains.

To test the Thalofide cell arrangement in its best spectral region the absorption band of liquid water at 1.μ was investigated. The band was found to be complex and some of its fine structure is indicated in Fig. 4, in which the absorption coefficient α is plotted as ordinate against the wave length as abscissa. α is defined by the relation $I = I_0 10^{-\alpha\kappa}$, where

I_0 is the initial intensity of radiation of a specified wave length and I the intensity after passing through x cms of water. The cell and apparatus of Fig. 2, with $E_2=72$ volts were used, and readings of the galvanometer could be repeated to within 1 mm. The slit widths were each 0.04 mm, or about 50 AU. The Thalofide cell arrangement was about 60 times as sensitive as the Hilger thermopile. Since the purpose of this paper is to call attention to the cell, no more will be said about the absorption bands of water. That will be left to a subsequent investigation.

The same apparatus was used at night with the infrared transmitting glass removed from the cell. The purpose of this experiment was to study the sensitivity of the cell in the visible. With the present apparatus it was possible to study the cell to 5000 AU. It was found that the cell was nearly uniformly sensitive from about 0.7μ to 1.2μ. The slit widths were 0.04 mm. From 7000 AU to 5000 AU the sensitivity decreased and in the vicinity of 5000 AU it was necessary to widen the exit slit to 0.1 mm in order to obtain a 100 mm deflection of the galvanometer. A slit width of 0.1 mm at 5000 AU corresponds to a spectral range of less than 50 AU showing that there was no loss of resolving power. The voltage E_2 applied to the cell was 90 volts.

It is possible to increase the sensitivity of the apparatus in several ways. Since the resistance of the cell is of the order of 100 or 200 megohms it is obvious that the resistance of the galvanometer plays no role. Therefore one method of increasing the sensitivity of the apparatus is to use a more sensitive galvanometer. In this respect the cell has an advantage over the thermocouple since with a thermocouple one has to use a galvanometer of high sensitivity but of very low resistance whereas for the cell the galvanometer resistance is unimportant. In all of the foregoing experiments the distance of the scale from the galvanometer was one meter which of course could be increased. Another way is to increase the voltage applied to the cell. All of these methods of increasing the sensitivity of the apparatus are limited by the steadiness of the galvanometer deflections. For infrared spectroscopy in the neighborhood of 1μ one might obtain increased resolving power by using the cell in conjunction with an echelette grating of about 20, 000 lines per inch.

In conclusion the author wishes to thank Dr. E. O. Hulburt for suggesting the problem and for his interest.

NAVAL RESEARCH LABORATORY,
WASHINGTON, D.C.
SEPTEMBER 24, 1926.

THE MEASUREMENT OF HIGH POTENTIALS AND A DESCRIPTION OF A NEW TYPE OF RESISTANCE

By Ross Gunn

ABSTRACT

Two methods of measuring potentials of the order of 100,000 volts are given which make use of ordinary laboratory instruments. The two methods are compared experimentally and good agreement is obtained to within 1 per cent at least. A new type of resistance with an "effective zero temperature coefficient" is described and its use for the measurement of high potentials is discussed. The methods are of particular value when the power source is a small one.

The measurement of potentials above 10,000 volts is often somewhat of a problem since the apparatus necessary to make such measurements is expensive and few laboratories can afford to make the necessary investment. Moreover, the power consumption is very large with the usual arrangements and this in turn requires a large and expensive source of high potential.

The writer has had occasion to make potential measurements up to 100,000 volts and has succeeded in measuring these potentials with a fair degree of precision, using apparatus found in almost any laboratory. It is believed that the method may have general interest, especially to those who have attempted to make measurements by means of the sphere gap.

The electrostatic voltmeter is, in general, the instrument preferred for ordinary use as it consumes very little energy and may be left permanently in the circuit. It must, however, be calibrated by some other more direct method in the position it is to occupy permanently. This is on account of the circulation of the oil brought about by the high potentials. Various writers have described high voltage electrostatic voltmeters; the most popular type seems to be a balanced dumbell swung on a bifilar suspension in the center of a large tank of oil. The axis of the dumbell is set so that it makes an angle of about 45 degrees with the line of centers of two other larger balls or disks placed two or three feet apart in the same tank of oil. The two large balls or disks are connected to the high potential side and the dumbell grounded.

The electrostatic voltmeter is placed in parallel with the calibration voltmeter and simultaneous observations made for a number of different potentials. The electrostatic voltmeter measures root mean square values and if rectified alternating current is used as a source of current

and this measured by means of a direct current instrument which measures average values; it will be necessary to limit the alternating component of the rectified current to a reasonable value. The difference between root mean square and average values amounts to ½ per cent for a "ripple" of 10 per cent. Thus a poorly filtered source may introduce considerable error and this difficulty must be carefully guarded against if the methods to be described are used to calibrate an electrostatic voltmeter.

The writer's source of high potential was a 5 K.W. 500 cycle generator and 100,000 volt transformer used with two kenotrons in the usual manner. A large filter system was used to smooth out the rectified current and at no time did the "ripple" exceed 5 per cent. The transformer was provided with an auxiliary secondary for use with an ordinary voltmeter and these voltmeter readings could be used as a rough check on the voltages determined by the methods to be described.

While working on this problem the writer used two methods to measure the high potentials and checked each against the other. The first method was a potentiometer method using a 1500 volt generator with standard voltmeter as the "standard cell." The second method, was essentially a voltmeter method using a precision shunt across the indicating galvanometer. Both methods required a high resistance capable of carrying at least 200 microamperes without appreciable change in resistance.

The resistances used to replace the very expensive wire resistances were made up of water which was made to circulate continuously through long uniform glass tubes. The continuous circulation guarantees uniform heating and replaces the warm water with cold with sufficient rapidity to keep the temperature rise of the water to small values. Sudden changes of resistance such as observed with the ordinary water or xylol-alcohol resistances when under load are entirely absent with this type of resistance. The intake and discharge ends of the tube were obviously maintained at ground potential since the water was taken from the city mains and discharged into the drain. The high potential connection was made to the mid point of the tube by means of a copper sleeve which served to join two adjacent sections of the glass tubing. These resistances were surprisingly constant under load and when proper arrangements were made to maintain the flow constant no change in resistance could be detected over periods of several hours. It was found desirable, however, since the resistance would change slightly from day to day, to calibrate them before and

after use. Cleaning solution was run through the tubing from time to time to clean off a deposit left by the city water which tended to decrease the resistance.

If sufficient current flows through the water to raise its temperature appreciably a correction must be applied which involves the temperature coefficient of the water used and the difference of temperature of the water between intake and discharge; due allowance being made for the fact that the room temperature is in general quite different from that of the water. This correction may always be made negligibly small by making the flow of water sufficiently large.

The resistance used in these experiments was made up of over 100 feet of 5 mm glass tubing which formed a two "wire" transmission line 50 feet long. The tubing was well insulated for high voltage by mounting on long glass supports. One end of the transmission line was grounded to the water pipe and drain while the other end was connected to the source of high potential which had its low side grounded. The resistance of this particular arrangement using city water which was circulated through the tubing continuously, was about 3×10^8 ohms. That is to say that the resistance of each "wire" was about 6×10^8 ohms. The power consumption at even 100,000 volts was not high, amounting to only about 40 watts, hence a powerful source of potential would not have been necessary for the operation of the voltmeter. This low power consumption is of great importance since the "ripple" in a rectifying set with a given filter system increases rapidly with the load and as pointed out this "ripple" may seriously affect the calibration of an instrument which measures root mean square values. Still lower power consumption would have been possible if distilled water had been used or the dimensions of the tube changed.

The connections for the potentiometer set up are shown in Fig. 1, when switch S is thrown to position P. Contact with the water at the points A, B, and C was accomplished by the use of copper sleeves which were used to join the various sections of the glass tubing together. The procedure is exactly the same as for the conventional potentiometer set up except that the ratio of the resistances of the total (AC) to the section (AB) is determined by two standard voltmeters in a preliminary measurement. A high range voltmeter together with a source of potential is placed across the whole resistance (AC) while a low range instrument with another source of potential is placed across the section (AB) and the system balanced by varying either source of potential. The polarities of each source is then immediately re-

versed and the system again balanced and the voltmeters again read. The average ratio of these calibrating potentials is obviously the ratio of the resistances and if the tube is uniform this ratio should be approximately the same as the ratio of the length (AC) to the length (AB). Experience has shown that the double determination of the ratios of the voltmeter readings or some similar procedure is necessary since polarization effects are not at all negligible when the potential drop over the section (AB) is less than 200 volts.

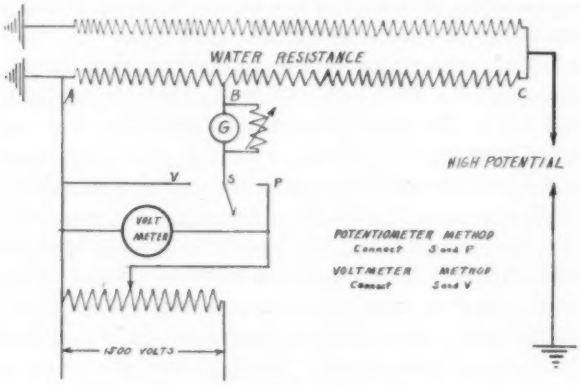

FIG. 1. *Circuit diagram.*

After determining the electrical ratio of the potentials the high range voltmeter should be shifted to the position shown in Fig. 1 and the unknown high potential placed across the whole resistance (AC). The galvanometer "G," which must be a rugged instrument properly shielded from all electrostatic forces, should be shunted and the potential across the section (AB) varied by some means until the galvanometer deflection is reduced to approximately zero. The voltmeter is read for this adjustment and the unknown high potential is at once determined by multiplying this reading by the ratio of the resistances determined by the preliminary measurements. No correction for change of temperature of the water is necessary in this case since all first order variations cancel out. This method is of particular value as a check method since the results are independent of Ohm's law if the tube is of uniform cross section.

To check the values of unknown potential as determined by the potentiometer method, a second method was used which seems capable of giving good results. This method was a simple voltmeter method

making use of a rugged galvanometer as an indicator which was shunted by a precision shunt. This combination was connected to the high resistance previously described as shown in Fig. 1 when switch S is thrown to position V. To calibrate this arrangement a known potential determined by a high range precision voltmeter was placed across the high resistance and the galvanometer in series, and the current measured by noting the galvanometer deflection and the position of the shunt. The voltmeter and power source are then disconnected and the high potential applied. The current is again determined by noting the deflection and the new position of the shunt. Obviously the unknown potential is equal to the ratio of the currents multiplied by the calibrating potential if the resistance has been kept constant.

If the flow of water is not sufficiently rapid the current flowing through the resistances may heat the water sufficiently to raise its temperature and change its resistance. Assuming that the increase in temperature is proportional to the distance from the intake and remembering that there are two resistances in parallel effectively and that the water is heated in both, one gets an expression for the correction in the following form:

$$E_{\text{true}} = E_{\text{observed}}\left[1+\frac{a(t'-t'')}{4}\right]$$

where a is the temperature coefficient of the water used as determined by experiment and is in general negative. $(t'-t'')$ is the difference in temperature of the water at discharge when the current is flowing and when the current is off, due time being allowed for equilibrium to be established.

These two methods were compared for a wide range of potentials. Table 1 gives a typical set of data and shows the type of agreement that may be expected. No correction for temperature change was necessary for this series of observations since the water was circulated with sufficient rapidity to keep the temperature correction to less than .3 per cent at 77,000 volts. The correction would be still less at the lower potentials.

The second method is probably the most convenient as errors due to polarization are not so liable to cause trouble and the potentials can be read directly without adjustment. This procedure would also be applicable to the measurement of alternating potentials if a carefully calibrated thermocouple or bolometer with suitable shunt is

substituted for the galvanometer system. Other modifications of the second method will suggest themselves immediately to the reader.

In general, alternating potentials may be measured most easily by means of an electrostatic voltmeter similar to the one just described. This electrostatic voltmeter is easily calibrated by means of rectified alternating current which allows the use of precision direct current standardizing instruments.

TABLE 1.

Voltmeter Method				Potentiometer Method		
E′ volts	Deflection	Shunt	E_{vm} volts	$E_{(AB)}$ volts	Ratio	$E_{pot.}$ volts
35	7.93	.001	36,870	714	51.42	36,700
45	9.95	.001	46,300	899	51.42	46,200
55	12.00	.001	55,800	1090	51.42	56,000
65	14.08	.001	65,450	1270	51.42	65,200
75	16.56	.001	77,000	1499	51.42	76,900

Calibration for Voltmeter. 21.5×10 cm deflection for 1000 volts.
Calibration for Potentiometer.

1118/22.0 = 50.75 1146/22.0 = 52.1 Average 51.42.

Calibrating potentials in one direction. Calibrating potentials in reverse direction.

The two methods described appear to be accurate to at least 1 per cent which is sufficient for most ordinary purposes. If the potential gradient in the water resistance is very large or there is other reason to suspect that the liquid does not obey Ohm's law the potentiometer method should be used since with uniform tubing this method is independent of Ohm's law.

It is the writer's belief that these simple means may be of some value in aiding workers on high potentials to obtain quantitative results of reasonable accuracy.

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METHODS FOR EXCITING AND FOR CALIBRATING TUNING FORKS*

By Elias Klein and Glenn F. Rouse

INTRODUCTION

The practicability of using tuning forks in accurate timing devices, for synchronizing mechanisms, and for the production of constant frequencies is at present receiving increasing attention. Unfortunately the descriptions of the methods for exciting and for rating tuning forks are scattered throughout the literature, some of which are not readily accessible to many investigators. It is the purpose of this paper to summarize a number of these methods and, in addition, to discuss the conclusions reached in repeating and testing some of them.

I

TYPES OF FORKS AND THEIR MODES OF EXCITATION

Although elinvar (a nickel-steel alloy), because of its negligible temperature coefficient of frequency,[1] seems to be an ideal substance to use for tuning forks, yet the expense and difficulty involved in working this metal make the production of such forks quite impracticable except in special cases. The same objections may be raised against quartz tuning forks. Besides, the elinvar forks show an appreciably greater change in frequency with change in amplitude as compared to steel forks.[2] Hence the present discussion will be concerned more directly with the ordinary soft steel forks which are maintained in vibration. Of course, much of what will be said applies as well to the special forks.

The type of fork that is desirable for precision and steadiness of operation (other factors which influence the frequency of a fork will be discussed later) is one in which the prongs are symmetrically balanced and which is mounted horizontally on a rigid heavy base. The balance of the fork is not as essential as the heavy mounting which can be made to reduce the antinodal vibrations of the stem to a minimum.[3] In general, the higher the frequency of the fork the more necessary it is to have a massive base.

To drive a fork continuously one of several methods may be employed:

1. The self driven electromagnetic make-and-break fork; that type in which the fork interrupts a direct current passing through an exciting magnet, usually placed between the prongs of the fork, and thus

* One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. See Editorial in this Journal *9*, p. 410, 1924.

sustains itself at or near its natural frequency, is too well known to need further comment here.

2. Alternating or interrupted current from an outside source passing through the electromagnet associated with a fork may be made to drive the fork provided the natural frequency of the fork is the same or an exact multiple of the frequency of the interrupter.[4] This method makes possible the driving of several forks by one interrupter fork, but its application is difficult because the ratio of frequencies between each driven fork and the interrupter is quite critical.[5]

3. A system of fork excitation far superior to either (1) or (2) is that in which a thermionic vacuum tube is made to drive the fork. This

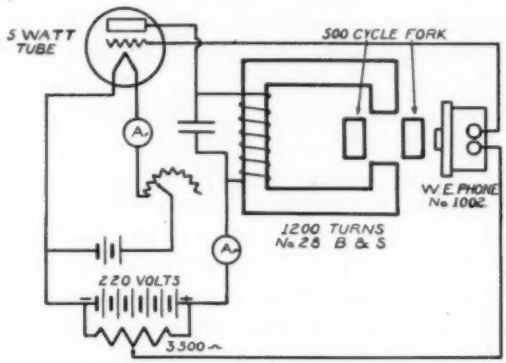

FIG. 1.

device has many advantages over the electro-mechanical drive. The fact that the vibrating elements of the fork are, in the vacuum-tube-drive, free from any mechanical attachments; that the vacuum-tube-fork can be maintained at frequencies beyond those of which the electro-mechanical forks are readily capable;[6] and, that it is a constant and reliable to a high degree of accuracy[7] make this mode of excitation very desirable. Moreover, the possibility of obtaining from a single tube-driven-fork[8] a large number of constant frequencies,[9,10] which may be used as secondary standards, shows the importance of this mode of excitation from the point of view of frequency measurement.

Several modifications have been made on the original method of Eccles[8] for driving a tuning fork by the use of a vacuum tube. Eckhardt et al[6] used a magnetic-reaction circuit somewhat as shown in Fig. 1, to drive a 500-cycle fork. In the original Eckhardt system the grid circuit was tuned and a grid coil wound on a laminated iron yoke was used in

the place of the telephone receiver, as shown. The writers set up a 384-cycle fork in accordance with Fig. 1 which functions very well. The location of the driving electromagnet as well as of the receiver magnets with respect to the length of the prongs of the fork is best determined by experiment, although Horton, Ricker and Marrison[10] have determined the desirable approximate location (see below). In this method, the fork does not start when the plate current is started through the driving magnet. The prongs must be pulled together and set in vibration.

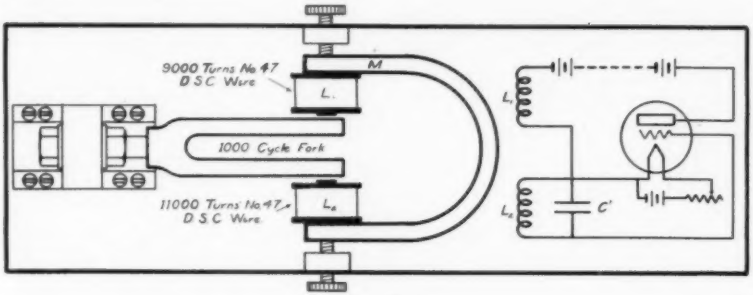

FIG. 2.

A self-starting fork of 1000 cycles was designed by Dye.[7] He used a polarizing field mode of excitation. The plate and grid coils were mounted upon the laminated polar projections of a polarizing magnet, *M*, as indicated in Fig. 2. A similar fork-generator of 2000 cycles was constructed at the Northolt Wireless Station,[9] England; and the 22nd harmonic was filtered and amplified for transmission, the frequency of which remained very constant.

It may be remarked that for a fork which is to be kept in a constant temperature enclosure or at a point which is not readily accessible to the observer, the factor of self-starting is of considerable importance.

A modification of the system shown in Fig. 1 for maintaining a 100-cycle fork was developed by Horton, Ricker and Marrison.[10] They designed a driving magnet which is, in effect, double that shown in the figure and which symmetrically surrounds the prongs of the fork. This magnet is located about two-fifths below the ends of the prongs of the fork, mounted vertically. Two telephone receivers, placed close to the prongs and about one third of the distance up from the base of the fork, were used to energize the grid of the amplifier. The use of this fork as a standard will be discussed later.

One of the simplest tube-driven forks with which the writers had occasion to work was designed by N. E. Bonn of the Leeds and Northrup Co.* This low frequency fork (50–70~) was devised for a synchronizing or speed control system. The schematic arrangements of the fork, as well as the fork itself, is shown in Fig. 3. The driving coil consists of an inductance, wound on a laminated magnetizable core, different portions of which are connected respectively in the grid and plate circuits of a vacuum tube. A capacity is shunted across the grid portion of the inductance.

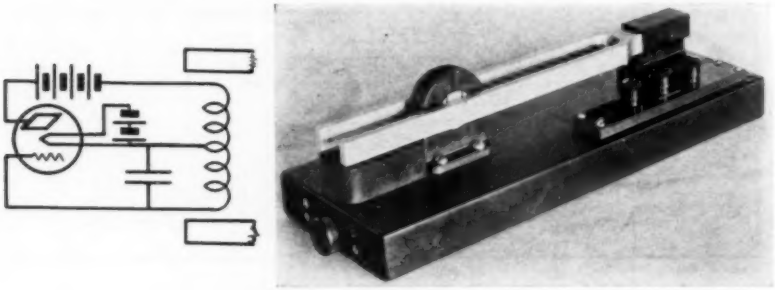

FIG. 3.

The mathematical analysis of a vacuum-tube-driven fork, similar to that used by Dye, was given by Butterworth[11] who pointed out the value of connecting condensers across the plate and grid coils respectively. More recently Hodgkinson[12] discussed mathematically the circuit of a vacuum-tube-driven fork without condensers. He showed also that for low frequency forks (50~) it is advantageous to employ transformers between the tube electrodes and the fork magnet coils.

Instead of energizing the grid electromagnetically, Pierce[13] used a microphone button to modulate the current in the grid circuit and thus drove a thousand-cycle fork by means of a vacuum tube. He modified a General Radio Audio Oscillator (fork-driven microphone) by mounting the microphone button at the base of the fork and otherwise adapting the system to a vacuum-tube drive.

4. Cady[14] has shown that a continuous note from a small size tuning fork may be produced by the piezoelectric action of Rochelle salt

* The authors gratefully acknowledge the courtesy of the Research Department of Leeds and Northrup Co., and the kindness of Mr. N. E. Bonn for the privilege of describing this fork and the speed control system discussed later.] The experiments with the Vreeland oscillator (see page 282) were performed in the research laboratory of this company.

crystals. A crystal plate is cemented endwise to each of the prongs of the fork near the tip. For a UT6 fork the Rochelle salt plates were $2\times1.1\times0.25$ cm. "It seems to make no difference whether the Rochelle salt plates are mounted with their smallest dimensions parallel to the prongs, or at right angles to them."* Each plate is provided with a pair of tinfoil coatings connected respectively in the grid and plate circuits of a vacuum tube working as a relay. Professor Cady has been able to excite a fork of lower frequency (UT3) but he finds that "it was much easier to drive the smaller fork." (UT6).

This mode of excitation is useful for small forks of higher frequencies (say above $1000\sim$) when it is desired to drive them temporarily or in the event that the electromagnets required for a vacuum-tube-drive are not available. For permanency of operation and for precision, the vacuum-tube-fork is preferable to the piezo-electric-fork. The crystal plates of the latter introduce a considerable change in the frequency of the fork, which of course, is determinable. But if for any reason the plates should have to be re-attached a new calibration of the system would have to be made. Furthermore, the lead wires to the crystal plates may influence the frequency of the fork in an indefinite manner.

II

FACTORS WHICH INFLUENCE FREQUENCY

Before proceeding with the methods of calibration it is essential thoroughly to appreciate the influence of certain factors upon the frequency of a tuning fork.

Amplitude. For electrically maintained, make-and-break tuning forks Dadourian[15] and later Wood[3] have shown that, for a constant spark-gap, an increase in amplitude of the fork, effected by a change of current in the electromagnet or a change in the position of the pole-pieces relative to the prongs, results in a decrease of its frequency. This is the same as for a "free" fork, one which is bowed and vibrates freely. However, for the electrically maintained forks in which the contact springs are liable to "overshoot" at contact, there is shown first a decrease in frequency with increasing amplitude then an increase in frequency for further increase in amplitude. To overcome this difficulty due to "overshooting" Wood and Ford[16] designed contact springs of a plunger type which can be adjusted and limited in their movements. By using these, Wood and Ford were able to obtain more constant frequency and greater reliability of operation.

* The authors are indebted to Professor Cady for some of the details in this write-up, which he furnished in private correspondence.

The amplitude of a vacuum-tube-driven fork can be changed only by introducing such changes in the grid and plate circuits of the vacuum tube as will of themselves vary the frequency of the fork and influence the amplitude only indirectly. Hence no systematic study has as yet been made on the variation of frequency with variations in amplitude solely. Nevertheless, small amplitudes are recommended[10] because of the small frictional and hysteresis losses.

Temperature. In general, variation in temperature is the most systematic and influential factor affecting the frequency of any fork. The values obtained by Dadourian, and Wood and Ford for electrically maintained forks; by Dye, and Horton, Ricker and Marrison for vacuum-tube-driven forks do not differ appreciably from the value accepted for free tuning forks, about 1.1×10^{-4} decrease in frequency per °C rise in temperature. This is due mainly to the change in Young's Modulus with temperature; the temperature coefficient of elasticity being about -2×10^{-4}, while the temperature coefficient of expansion is of the order of 10^{-5}.

Damping. Damping per se does not cause any appreciable[3] change in frequency of a tuning fork. But the variations in the electrical constants of a driven fork may affect the damping as well as the period of the fork. Therefore it is desirable to eliminate the damping of the fork to the greatest possible extent. From the study of the vacuum-tube-driven fork Dye concludes (a) that the damping of the fork must be as small as possible, (b) that the dynamical energy associated with the motion of the fork should be large compared with the electrical energy of the circuit (i.e. damping of electrical circuit should be large).

Weight of Base. The frequency of a fork, whether excited by an electromagnet or a vacuum tube, increases slightly for increased weight of base. Wood concludes that "it is desirable therefore, that a portable fork, whose frequency is to remain constant in all positions should have a heavy rigid supporting base which serves both to reduce the amplitude of the stem and to reduce the coupling with external systems."

Position of Prongs. A measurable effect on the frequency of a fork is caused by a change in the position of the prongs of the fork. For example, a fork vibrating in a vertical plane against the action of gravity shows a definite positive change in frequency from that when the prongs are vibrating in a horizontal plane.[3,7]

Constants of Vacuum-Tube Circuit. The investigations of Dye[7] and of Horton, Ricker and Marrison[10] show that changes in plate voltage and filament current cause but slight changes in the frequency of the

fork; that small variations in the capacity across the grid-coil introduces a negligible change in the frequency; that aging of the vacuum-tube which drives the fork shows no observable variation in the frequency; that if a polarizing field is used, it must be held constant to 1 per cent when great accuracy is desired.

III

INDICATING INSTRUMENTS FOR THE CALIBRATION OF FORKS

Forks having sufficiently high pitch may be compared acoustically by the method of beats. However where the comparison is made between a fork and another sounding body of slightly different quality, or between two forks of low frequency, (say below 100~), the sound-beat-method is difficult and inaccurate.[17]

Since the satisfactory methods of determining the frequency of a vibrating body depend upon devices which record pitch automatically or whose indications can be observed visually, it seems advisable to list a few such devices. These instruments will, of necessity, quite often involve a mechanical vibrating element. It should be noted that any device, in which the vibrating element has a natural frequency of its own and hence responds to some impressed frequencies more readily than to others, cannot be used in any large portion of the audible range with uniform success. These instruments are best suited for a limited region of frequencies. When tuned they are selective and are unaffected by harmonics. Among such devices are the vibration galvanometer;[17] the string galvanometer;[21] the Phonodeik;[18] the Phonelescope[19] (a modified telephone receiver) and certain other types of oscillographs. A device not at all limited in its frequency range is the cathode ray oscillograph.[20] Tests have shown that it can be recommended above all other instruments, used in the comparison of frequencies, for its adaptability and for the accuracy of results obtainable. However the cathode ray oscillograph is not as well suited to photographic records as are other types of oscillographs. In using any one of the above mentioned instruments as a device for comparing frequencies, it will be found best to place it in a circuit which is inductively coupled to the circuit containing the fork to be calibrated and to the circuit containing the fluctuating emf of known frequency, as shown in Fig. 4. It is desirable that the mutual inductances M_1 and M_2 be variable and that these be coupled as loosely as possible to avoid the interaction of one circuit upon the other. The respective magnitudes of the emf's induced in the instrument circuit should, when feasible, be made

equal by the variation of M_1 and M_2. As the frequencies of the two induced emf's approach equality or integral multiplicity a beat effect is noted on the instrument, and its rate can be accurately determined. If the frequency of the standard source is variable it is often preferable, (in the case of the string or vibration galvanometer) for visual observation, to time slow beats rather than to set the two sources in synchronism. But when the indicating instruments shows clearly

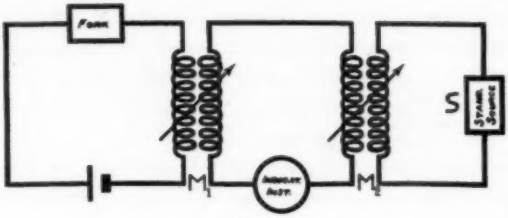

FIG. 4.

defined Lissajous' figures, as in the case of the cathode ray oscillograph, the accuracy is not impaired by noting the frequency at which stationary figures are observed. Of course, still greater accuracy may be obtained by counting beats for frequencies of the standard just below and just above the unknown frequency.

IV

METHODS OF CALIBRATION

The methods for rating forks may be arbitrarily divided into two general classes, namely; approximate, and precise. The former includes such arrangements as will yield results accurate to about one per cent, and which are more or less indirect. In the second group are enumerated methods by which the highest degree of precision is attainable, say, at least to 0.01 per cent. The various methods usually given in elementary texts on sound will not be included in the following list. Reference to these will be found in the bibliography.[42]

A. *Approximate Methods.* 1. With a Phonic Wheel Stroboscope. One of the simple yet reasonably accurate methods of calibrating a tuning fork is by the aid of a phonic wheel used stroboscopically. The phonic wheel or synchronous motor, invented independently by La Cour[22] and Raleigh,[23] consists of two electromagnets and a rotating wheel-armature with an even number of iron teeth equally spaced on its circumference. Dodourian[15] finds "that the wheel runs with greater smoothness between two than between four pole-pieces." Co-axial with

the wheel is attached a light metal disc in which one or more radial slots are cut at equal intervals on a circle concentric with the disc. Several such circles of various radii with different numbers of slots may be arranged in this manner. A well adapted design of this instrument made to synchronize with electric forks of the make-and-break type, below 300 cycles per second is manufactured by Pye and Co., England.

For forks of higher frequencies and having small power output, e.g. the vacuum-tube-driven fork, the wheel when driven synchronously by the fork must be carefully balanced and so mounted that it requires an inappreciable amount of power to keep it running. The authors have constructed a phonic wheel to synchronize with the vacuum-tube fork after the manner of Dye,[7] with slight modifications. It consists of an aluminum wheel, about 11 cm in diameter on which is forced a steel rim about .4 cm thick. Twenty teeth and spaces are cut equally and at equal intervals on the rim. Twenty radial slots are cut in the aluminum disc. It is mounted on a hardened steel spindle which is made to run in jeweled bearings. A pair of W. E. 1002 telephone receivers are so adjusted that diametrically opposite teeth are simultaneously passing between the polarized magnets of the phones. As a synchronous motor this wheel, after it is run up to speed, which may be accomplished by a blast of air against the teeth of the wheel continues for hours. The authors adapted this method of turning the wheel up to synchronous speed in preference to the scheme used by Dye[7] because of the difficulty they encountered in reproducing and maintaining in motion the vacuum-tube-driven motor of Eccles and Jordan[41] which Dye utilized. He first set up the wheel as a vacuum-tube-driven motor which, by the aid of a two-stage amplifier was run up to a speed of over 3000 rpm. Then by gradually reducing the self-driving power of the wheel and slowly applying power from the tuning fork (through a three-stage amplifier) the wheel was pulled into synchronism with the fork.

Mr. Dye was kind enough to suggest possible means of overcoming the difficulties with the vacuum-tube motor, but the authors find that a blast of compressed air from a constricted glass nozzle directed tangentially against the teeth of the wheel is simpler than the motor, and works admirably well. Mr. Dye also stated "I do not think the phonic wheel considered as a motor is sufficiently stable to be of much value as a steadily revolving piece of mechanism." The authors have corroborated this statement.

For calibration, the fork is set behind the slotted disk or wheel and one of the prongs, or a light stylus attached to the prong, is observed

thru the slots of the revolving wheel. When synchronized the prong appears at rest. From the number of slots and the speed of the wheel the frequency of the fork can be calculated.

Instead of the fork driving the wheel the latter may be attached to the shaft of a variable speed motor and synchronized with the fork stroboscopically. By knowing the speed of the motor a rough approximation may be had of the fork period. Conversely, the speed of the motor may be determined quite accurately if the frequency of the fork is known.

2. By Means of a Precision Galvanometer and a Standard Condenser. This method, which was originally developed to measure capacity, will be found useful in case it is desired to calibrate a contact-driven-fork. With reasonable care the method is easily accurate to less than one per cent.

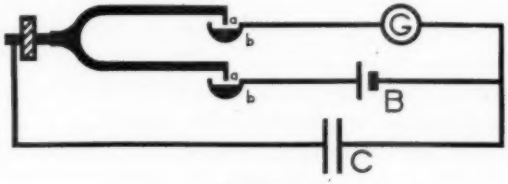

FIG. 5.

The calibration may be carried out as follows. Two fine and light but stiff wires, a, are attached to the prongs of the tuning fork. The two small mercury cups, b, (hard-rubber rods partly drilled), are adjusted to such a position that when the fork is not vibrating the wires, a, clear the mercury surfaces by at least .5 mm. The circuit is completed as shown in Fig. 5. G is a galvanometer for which the figure of merit is accurately known, C is a precision air condenser, B is a battery whose emf has been carefully measured. Care must be taken to prevent leakage.

Because of the way in which a fork vibrates, when contact is made at one mercury cup the circuit through the other cup is open. The condenser, C, is therefore charged n times per second and discharged through the galvanometer at the same rate. The galvanometer will show a steady deflection proportional to the average value of the current provided the period of the galvanometer is large compared with the interval between discharges. The value of n is obtained from the equation $n = Fd\ 10^6/CV$ where F is the figure of merit of the galvanometer in amperes per division; d is the deflection in scale divisions;

C is the capacity of the condenser in microfarads and V is the emf of the battery in volts.

3. Beat or Null Method by a Variable Speed Commutator. In this method a circuit containing one of the indicating instruments previously mentioned, is coupled to two other circuits, as shown in Fig. 4. The voltage of known frequency impressed on circuit S, may be generated by means of a commutator whose angular speed can be accurately determined. If this arrangement is to prove successful the commutator must be susceptible of very fine adjustment of speed and its constancy of rotation should be controllable.

Instead of generating the fluctuating voltage by means of a commutator a small emf can be obtained without the use of any brushes whatever, by placing the pole pieces of a telephone receiver so that they barely clear a series of equally spaced iron teeth on the rim of a revolving disk, similar to the high speed phonic wheel previously described. This latter arrangement has been found to give an appreciable deflection on the cathode ray oscillograph by using only one receiver and a high frequency transformer at M_2.

4. Beat or Null Method with Vacuum Tube Oscillators. Perhaps the simplest and most widely used source of alternating current of any desired frequency is the vacuum tube generator. The output leads of some such generator when connected at S, will furnish an idea of the frequency sought. But for quite accurate results the generator must satisfy more rigorous conditions. Of the large number of different circuits used for vacuum tube oscillators, there are relatively few in which the frequency will remain fixed when once adjusted. Considerable change in frequency is produced by slight variations in the filament current and plate potential. Moreover, unless special precautions are taken to eliminate the many harmonics which are present in the generated oscillations great confusion arises when such an oscillator is used as a standard of frequency in a beat or null method.

The Western Electric 8-A Oscillator proved very satisfactory for the purpose at hand. It covers a range of frequencies from 100 cycles to 50,000 cycles per second and with reasonable attention any frequency in the range can be maintained constant to one per cent. This four tube oscillator (and amplifier) is quite elaborate and expensive. However Horton[24] of the Western Electric Company has designed a single tube oscillator, in principle similar to the 8-A, which should furnish constant frequency and good wave form for small output. Horton's original paper on "Vacuum Tube Oscillators" should be consulted to appreciate the features of this circuit and its limitations.

A constant frequency oscillator (1000~), using two tubes, has been devised by Gunn[25], who claims "that the frequency variation was less than one-tenth of one per cent for a plate voltage change of 50%, and a change of three-tenths per cent for 43% change in filament current." This oscillator should also be free from the disturbing effect of harmonics.

A remarkably constant fixed frequency and its harmonics may be obtained from a vacuum-tube circuit in which a piezoelectric quartz plate is inserted between the grid and filament or between the grid and plate.[12,16] Such an oscillator, because of its very high frequency, is not suitable for direct comparison with tuning forks, unless the upper harmonics of the tuning forks be used. However, Pierce[13] has shown that upon the frequency of a piezoelectric oscillator may be superposed the frequencies of an electrical system which is in the same vacuum tube circuit as the piezoelectric plate. The beat-notes between these two frequencies may, by adjusting the capacity and inductance of the electrical system, be used in the audio-frequency range as standards against which to calibrate tuning forks.

Another method for producing audio-frequency oscillations by means of a quartz plate and a single vacuum tube was developed by Hund[26]. This is accomplished by grinding a very small step in a parallel plate, which then has two thicknesses. Two high-frequency vibrations are thus produced simultaneously. The beat-note of these coexisting frequencies may be used as a standard audio-frequency.

5. With a Neon Lamp Stroboscopically. The phenomenon of "Flashing," or intermittance, in neon discharge-tubes may, very conveniently, be utilized as a source of variable frequency with which to compare tuning forks stroboscopically. This property of neon glow lamps was first observed by Pearson and Anson[27] in the case of the "Osglim" lamp. The latter consists of two pure iron electrodes separated by a few mms and contains a mixture of 75 per cent of neon and 25 per cent of helium at a pressure of about 10 mm of mercury. A lamp of this type, when shunted by a condenser, the combination being in series with a high resistance and a battery, as shown in Fig. 6a will flicker at regular intervals of time. The periodicity of this intermittent source depends upon the capacity and resistance of the circuit when the voltage is constant. For a given capacity and voltage there is a critical resistance below which the discharge becomes continuous. For a constant voltage and a fixed resistance the period of flashing varies linearly with the capacity to a fair degree of precision. Similarly,

when the voltage and capacity remain fixed the period varies directly with the resistance.

The authors were unable to procure an "Osglim" lamp, but a lamp known as "Osram" was kindly loaned us by Dr. Rudy of the Nela Research Laboratories through the courtesy of Dr. W. E. Forsythe. This lamp showed characteristics similar to those described by Pearson and Anson,[27] and Taylor and Clarkson[28] for the "Osglim" lamp. With a constant resistance of .4 megohm and a battery of 120 volts, the period of the "Osram" lamp varied linearly with the capacity quite accurately for frequencies up to one hundred per second. The

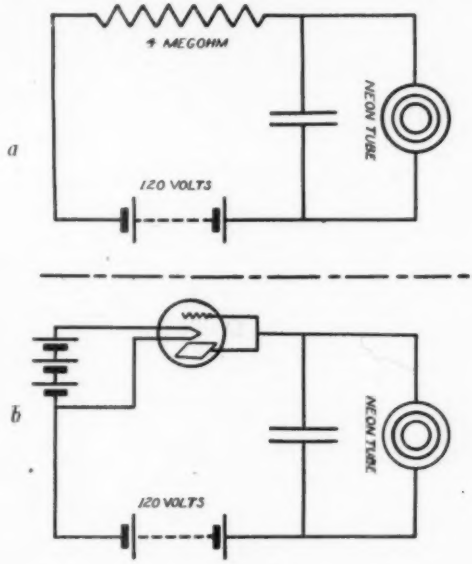

FIG. 6.

lamp functions best when the periodic glow does not continually shift from one part of the electrodes to another, and works remarkably steady after it has been run for some time.

Considerable resistance and capacity are necessary for the above described neon lamp if it is to cover a wide range of frequencies. When the requisite large amount of resistance is not available, a two electrode thermionic tube may be substituted for the resistance, as shown in Fig. 6b. The thermionic valve must be run at saturation in order that the charging current to the condenser shall be constant. Kipping[29]

states that "the frequency is preferably controlled by varying the capacity of the condenser, but alternatively the filament-heating current of the diode may be varied. It is essential that the current through the diode be the saturation current of the latter throughout the charging of the condenser."

A series of experimental curves showing the relation between the period of flashing and the capacity, for different filament currents, have been obtained with a V.T. 14 tube in which the grid and plate were connected together. These curves show a very smooth linear relation. By a careful potentiometer adjustment, the filament current may be regulated so as to control the period of the lamp and thus replace not only the resistance but also some of the capacity.

6. By a Vibrating String Stroboscope. The method devised by Guillet[31] for measuring the speed of a rotating disk stroboscopically

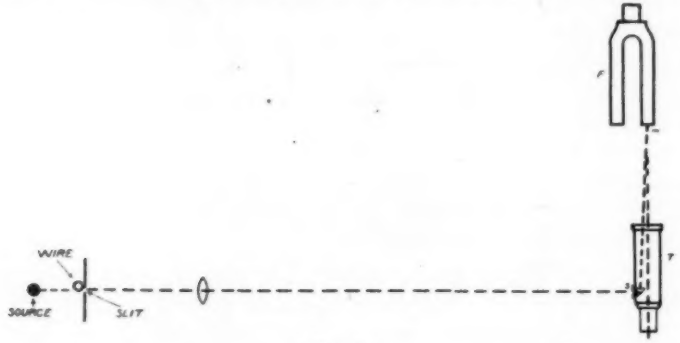

FIG. 7.

can, by slight modification, be used for the determination of the frequency of a tuning fork. The adapted scheme may be somewhat as follows. A beam of light passes through a slit in a small aluminium disc attached to a steel piano wire. The slit is parallel to the wire which is mounted vertically. The light from the slit is focussed on a totally reflecting prism, Fig. 7, located in the focal plane of the eyepiece of an Abbe autocollimating telescope which, by virtue of slit s' in front of the prism (not shown) serves both as a collimator and observing telescope. After reflection at the prism the light is directed along the axis of the telescope, leaving the objective in parallel rays. These rays strike the polished end of the prong, or a small mirror cemented to the prong, normally returning to form an image of the slit s' in the focal plane of the telescope. When the wire and fork are set in vibration

a series of flashes are observed in the telescope. The frequency of the wire is now adjusted until distinct coincidence of the stroboscopic images are observed in the telescope.

By using a pendulum which controlled a reflected beam of light instead of the wire as above described, Reed[33] was able by this method to obtain an accuracy of about one part in 10,000 when the fork was shielded from external temperature effects.

The vibration frequency of the string can be regulated within rather wide limits, by varying the tension and by changing the distance between the knife-edges supporting the string. With a steel piano wire (approximately 0.026 cm diameter) mounted horizontally, Jolliffe and Hazen[34] found the sonometer to have a range from 80 to over 3000 cycles. Guillet adjusted the tension of his wire-stroboscope by turning a graduated drum which was calibrated to read the frequency directly.

Another means of applying this monochord in the determination of tuning fork frequencies is by attaching to the wire a small mirror, instead of a slit, and setting it up against a tuning fork of unknown frequency, as in the well known manner for the production of Lissajous' figures.

For work of moderate accuracy the vibrating string or monochord is a very convenient source of variable frequencies. This is so because of the ease and readiness with which it is set up and adjusted to the desired frequency which may be calculated in advance. However, the the vibrating string is extremely rich in harmonics hence due care must be exercised in using the vibrating string as a source of standard frequency.

From the investigations of A. Guillet[31] and A. Bertrand[32] on the methods of measuring the speed of rotating machinery, there results another device applicable to the study of tuning-fork calibration. It consists of an electrically maintained vibrating string which controls the flashing-period of a neon lamp. By the adjustment of tension previously described, this scheme is susceptible of variations in frequency over a wide range and, with the neon lamp, forms a simple stroboscopic arrangement. The essential features of the instrument are shown in Fig. 8. A 12-volt battery is connected across a circuit containing a control rheostat, a contact point, the vibrating string and two driving coils. The driving contact is placed near a point of minimum amplitude on the string so that it will introduce no perturbations in the true period of the string. The driving coils are near the center of the string. As the current increases and decreases in these coils

alternating magnetic forces act on a small magnet, placed on the string, and keep the string in vibration. By means of wires, a, fastened to the string, the primary circuit of an induction coil is made and broken as the string vibrates. The secondary of the induction coil is across a neon lamp which, as the string vibrates, flashes with a frequency exactly equal to that of the string.

To compare the frequency of a fork with that of a neon lamp or the Vreeland Oscillator (discussed later), means must be provided for viewing the stroboscopic images which are produced. This is accomplished either by the aid of a phonic wheel run synchronously by the fork or, by a shutter arrangement attached to the fork. The shutter consists of two light metal plates mounted close together near the ends

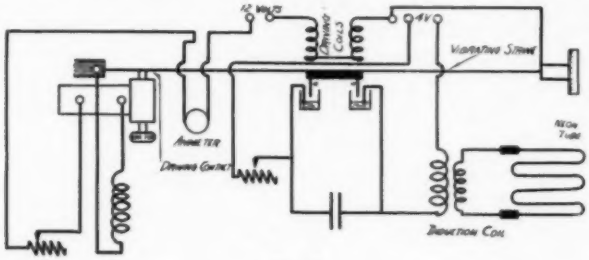

FIG. 8.

of the fork-prongs, parallel to the plane of vibration. These are adjusted so that a slit-opening is formed between them when the prongs are at the extremity of their outward swing. On looking through this opening, or through the slit in the phonic-wheel disc, intermittent light will be observed if the frequency of the lamp is nearly the same or a submultiple of the frequency of the fork. Upon increasing or decreasing the capacity across the lamp, in small steps, the period of intermittance will gradually increase until a steady darkness (or continuous light) will appear. At this point the two sources are in synchronism. When the lamp-frequency is a large multiple of the frequency of the fork, it is impossible to observe synchronism (in the manner indicated above) between the two vibrating sources with a finite slit-opening. Under these circumstances, the time between flashes of the lamp is small compared with that for which the fork shutter remains open; that is, the lamp will complete a number of cycles during the time the slit is in the line of sight. Hence, due to the persistence of vision, the light will appear continuous when viewed through the

opening in the fork-shutter, and no synchronism will be detected. Therefore, in visual stroboscopic comparisons it is advisable to use the standard lamp source at frequencies which are submultiples of the unknown fork.

The frequency of the fork may be calculated from the expression

$$n = 2(N' - N)$$

where n is the frequency of the fork; N' is the frequency of the lamp at any synchronizing submultiple or harmonic of the fork; and N is the lamp-frequency at the next successive harmonic, higher or lower.

B. *More Precise Methods.* 1. The Calibration of Tuning Forks by a Pendulum of a Clock. Since the accuracy of a reasonably good clock is about one part in 100,000[30] when timed for several hours; and since only a small amount of auxiliary apparatus is required for the comparison of a tuning fork with a primary standard like a clock, this method of calibration is of great value and very desirable.

The first to use the pendulum of a clock for the rating of a tuning fork was Rayleigh,[37] who devised an exact stroboscopic method. He concentrated a beam of light on a small silvered bead which was carried by the pendulum and which was observed through a shutter attached to the fork. Later[38] he used a fork-driven phonic wheel whose armature carried a disc with a single slit through which the illuminated bead was viewed. With either arrangement, light from the bead is seen in a number of positions of the pendulum, which depends on the ratio of the frequency of the fork to that of the clock. If this ratio is an integer, the positions will appear stationary; if not, the positions will seem to change slowly. To avoid confusion only one of these positions is observed by placing a fixed narrow vertical slit in front of the pendulum. Generally, the spot of light will now appear to move slowly across the slit. If the apparent motion of the spot is in the same direction as that of the pendulum, the latter is gaining on the fork; if the opposite is true, the fork is gaining on the pendulum. By observing the time interval between successive appearances of the spot of light, the exact period of the fork can be calculated provided the frequency of the fork is approximately known.

Comparisons of fork-frequency with that of a pendulum have been made chronographically by Dadourian[15], and by Wood and Ford[16] for electrically driven forks; by Dye[7], and by Horton, Ricker and Marrison[10] for vacuum-tube maintained forks. The usual arrangement for this method is shown in Fig. 9. The tuning fork synchronously controls a phonic wheel which drives a gear or worm-wheel of the proper ratio n

so as to close a contact every nth vibration of the fork. This contact, which is made for as short a duration as possible, is registered chronographically alongside of the automatic record of the rate of the standard clock or pendulum.

Ferrie[35] has shown that the frequency of a maintained tuning-fork can be measured by the aid of a pendulum whose impulses are recorded by means of a photoelectric cell. Ferrie and Jouast[36] devised a method of using the photoelectric cell to maintain also the pendulum giving rise to the impulses. This method is very similar to that described in (2) below.

A paper from the sound section of the Bureau of Standards, to appear in J. Frank. Inst. Feb. 1927, will discuss the method of rating a tuning fork by a pendulum to a high degree of precision.

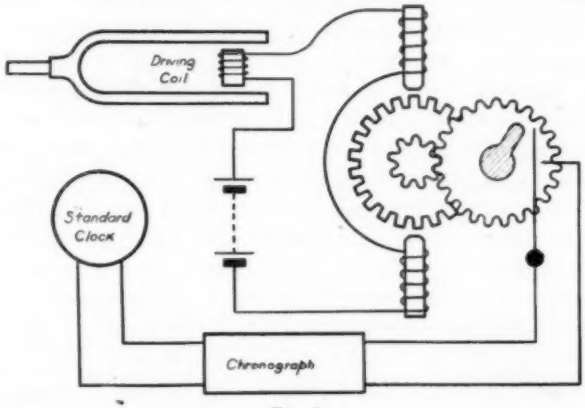

FIG. 9.

2. A Clock-Controlled Tuning Fork. Instead of calibrating a fork against a clock, the latter may be used to control the frequency of a fork to a high degree of precision. This scheme has been thoroughly worked out by Ferguson[30] for an electrically driven fork. The fundamental idea of this control is that each half swing of the pendulum of the clock shall affect and correct the divergence of time between the interval as measured by the clock and that required by the fork to complete a given number of vibrations. The method of control can best be understood by reference to the schematic diagram, Fig. 10. A 50-cycle fork operates the contact marked "fork", in a manner similar to that shown in Fig. 9. This contact closes once for every 25 cycles of the fork. The contact marked "clock" is made every half second by

clock. When the clock contact closes, relay R_1 operates and locks until released by the short-circuiting of its windings at the closing of the fork contact. While the relay is operating, the condenser C is charged through the resistance r_1 by the battery B_1. The voltage of this battery is such that when applied to the grid of the vacuum tube it will reduce the space current to zero. The condenser C continually discharges through the resistance r_2. The mean potential of the condenser, applied to the grid of the vacuum tube, modifies the space current which is passing through the damping coil, between the prongs of the fork. A stable condition exists when the condenser-discharge each second

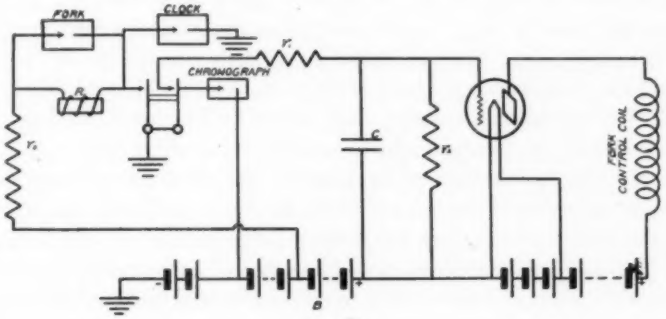

FIG. 10.

equals the charge. This state obtains when all periods of contact of the relay are of equal length. Any increase or decrease in the period of contact of the relay affects the mean potential of the grid and changes the space current, that is, changes the damping effect on the fork. Since the opening of the relay is brought about by the closing of the fork contact, the space current of the tube is thus controlled by the fork, and the instant the frequency of the latter shows any deviation from the mean value, forces are set up by the controlling coil which tend to hold the frequency constant.

For the particular arrangement just described, it took about 15 minutes for the fork to come into proper phase relation with the clock. Chronographic records show that when this phase relation is once established there is practically no hunting.

In order to obviate the error, which often occurs even in excellent clocks, due to a cyclic variation in the length of time between successive impulses of the escapement wheel, the clock-control contact was taken directly from the pendulum instead of the escapement wheel. Light

falling on a photoelectric cell was cut off by the pendulum as it passed through the middle of its swing. The periodic flow of photoelectric current, impressed on the grid of a vacuum tube, actuated the pendulum by a series of relays and a control coil. A steel bar carried at the lower end of the pendulum moved into the coil, and the driving impulse was one of attraction between the coil and the bar.

With this method of control it is possible to obtain a fundamental standard frequency over a short or long period of time, constant to one part in 100,000, or at least as accurate as the clock to which the fork is synchronized. Furthermore, it is suggested that with a good compensated pendulum, in a vacuum, having no mechanical connections, this method should yield the highest attainable accuracy.

An arrangement similar to the above was set up by Mr. C. H. Presbry and one of the authors using an ordinary laboratory clock, the period of its pendulum being two seconds. The clock contact was obtained from the pendulum by means of a Kunz photoelectric cell. No time lag was experienced in the action of the photoelectric cell or the vacuum tube (101D) associated with it. A 75-cycle fork was kept in synchronism with the clock for a whole afternoon.

3. With a Vreeland Oscillator. Morgan and Lammert[39] have shown that the frequency of a Vreeland oscillator remains quite constant after it has been run for several hours, hence the flicker of this type of oscillator should furnish an accurate means for the rating of a tuning fork stroboscopically. The method of calibration and the arrangement of the fork relative to the oscillator-bulb is precisely the same as previously described for the neon lamp. However, since the mercury arc of the Vreeland oscillator is a larger and a more intense source of light than the neon lamp, it is advisable to place a narrow slit in front of the upper portion of the bulb. Furthermore, care should be taken not to confuse the occasional flash due to the sputtering of the mercury arc with the periodic flicker which is the effect of the deflecting coils, in the oscillatory circuit, upon the column of ionized mercury vapor.

Although the ordinary laboratory type of Vreeland oscillator, Type "*E*", has but two frequency-settings, 500 and 100 cycles, yet the frequency can be varied from about 100 to 4000 cycles per second by properly adjusting the capacity and inductance associated with the oscillatory circuit. By adding sufficient inductance and capacity to the proper external taps of type "*E*" oscillator, the authors were enabled to show fundamental synchronism with a 60-cycle fork, stroboscopically.

4. By the Aid of a Fork With Special Vacuum-tube Circuits. The far reaching possibilities of accurately controlling the frequencies produced by means of thermionic tubes should not be overlooked in the study of tuning forks. It is not within the scope of this paper to discuss these possibilities. Nevertheless, mere mention will be made of one very ingenious application of vacuum-tube circuits to the production of standard frequencies. Perhaps this will suggest the feasibility of a continuously variable source of frequency covering the range of tuning forks. For the rating of precision forks this type of source is still wanting.

The scheme referred to above was developed by Horton, Ricker and Marrison[10] for the generation of frequencies ranging from 100 cycles up to 1,000,000 cycles. It is briefly as follows: Certain harmonics of a 100-cycle vacuum-tube-driven fork (previously discussed) were combined so as to produce frequencies equal to the sum or to the difference of the frequencies of any two of the harmonics generated. The current of the resulting frequency was amplified. By modulating the output of this amplifier with that of a specially built oscillator, frequencies accurate to the nearest cycle were obtained over a considerable range. Very high accuracy is claimed for this arrangement.

5. With a Constant Speed Rotor. There are many experimental arrangements for calibrating tuning forks which require a variable speed motor. However, since it is not practicable to regulate a variable speed motor to an accuracy of 0.01 per cent, a rotor of fixed speed controlled by a tuning fork is employed to advantage. The speed constancy of this rotor is maintained to the same degree of precision as is the frequency of the tuning fork. For a vacuum-tube-driven fork the accuracy claimed is as high as 1 part in 100,000.

Assuming for the present that such a constant speed rotor is available, its applications to the problem at hand lie mainly in stroboscopic methods. Some of these will now be described.

(a) On the axis of the rotor is mounted a disc which is provided with a series of cocentric circular rows of equidistant dots. Each circle may have its own color and its own interval between dots. The surface of the disc is well illuminated and placed behind the fork under investigation. If the fork is not provided with a shutter-adjustment, the slit of a phonic wheel, controlled by the fork, should be located in front of the disc and in line with a chosen circle of dots. When the disc is rotating and the fork set in vibration it is possible to find some circle of dots which, through the shutter-opening, appear stationary. From

the speed of the rotor and the number of dots in the particular circle, the frequency of the fork can be calculated.

(b) To obviate the necessity of loading the fork either by a shutter or by a synchronous motor, the disc is provided with concentric circular rows of holes instead of dots. On the polished end of one prong is concentrated a beam of light. If the area illuminated is large compared with the holes in the disc, a narrow slit should be placed between the illuminated prong and the revolving disc. The prongs of the fork will appear at rest when viewed through a given set of holes.

(c) Rayleigh's stroboscopic method for speed regulation[40] can, with the aid of a constant-speed rotor, be adapted to the calibration of tuning forks.

Upon a white flexible paper are drawn horizontal rows of alternate black and white rectangles. The height of the rows are equal, but the base of the rectangles are gradually diminished from row to row. The edges of the drawing are finished so that when the paper is mounted on a drum, the rows will form reentrant circles of black and white rectangles. The drum is mounted on the shaft of the constant speed-rotor.

Viewed through the fork-shutter, there will be observed some circle of revolving rectangles which appear nearly stationary. Usually two contiguous circles can be found which seem to revolve slowly in opposite directions. Beats may thus be accurately timed. From the number of black or white rectangles in the given circle, the speed of the rotor and the number of beats per second, the frequency of the fork is easily calculated.

With regard to the rotors, three kinds suggest themselves whose speeds are of the required constancy. The first is the ordinary phonic wheel synchronously driven by a vacuum-tube-fork. The difficulties of this motor have been mentioned before. The second type is a synchronous motor which is likely to be more stable and better suited for continuous operation than the phonic wheel. It is the design of Horton, Ricker and Marrison, who used this kind of motor in the comparing of the frequency of their vacuum-tube-fork with that of a standard clock. It differs from the phonic wheel in that its stator coils, ten in number, are separately supplied by direct current and that the alternating current from the fork-circuit is passed through a fixed stator coil within which the axis of the rotor revolves.

A constant speed rotor in the form of a motor-generator, controlled by a vacuum-tube-fork, was devised by N. E. Bonn of Leeds and

Northrup Co. The authors made several tests on this machine and could detect no perceptible change in the rotor-speed when the field current of the motor was reduced considerably below its normal value; and, when a friction load was suddenly applied to the rotor-shaft. The observations were made stroboscopically, against a Vreeland oscillator. A complete description of this device will be published shortly by its inventor. A schematic diagram Fig. 11, of this arrangement is published with the permission of Mr. Bonn.

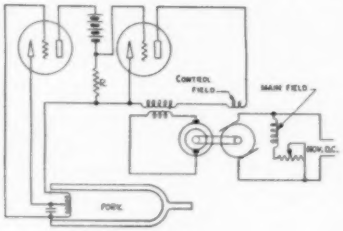

FIG. 11.

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